Year 8 - Mathematics . . .

Milestone Assessment 1

Speak Like a Mathematician

Balance - The amount of money in a bank account (in financial questions)

Credit - Money coming into an account

Debit - Money leaving an account

Negative - A value less than zero; has a minus sign

Factor - Non-decimal values that multiply together to get another value

Highest Common Factor (HCF) - The biggest factor two or more numbers share

Multiple - Found by multiplying any integer by positive integers

Highest Common Factor (HCF) - The biggest factor two or more numbers share

Base - The number that gets multiplied by a power

Index - (Indices) The number that tells you how many times to multiply the base by itself

Standard form - A system of writing very big or very small numbers

Significant figure - The most important figures (digits) to signify the size of the number

Approximate - To estimate a number, amount or total by rounding

Solve problems with finance

Profit = Income - Costs

Credit – Money coming into an account

Debit – Money leaving an account

Money uses a two decimal place system
14.2 on a calculator represents £14.20

Check the units of currency – work in the same unit

Directed numbers

+ + → +

− − → +

+ − → −

− + → −

eg a = -5 and b = 2

$a^2 = a \times a = -5 \times -5 = 25$

b + a = 2 + -5 = **-3**

Subtract directed numbers

= −1
= 1
Representations

Representation for calculation

2

"Subtract" – means take away or remove

2 − − 1 = 3

Take away one

Start with the representation of 2

= 3

2 − − 3 = 5

= 5

Generalisation

− − = +

Multiply/ Divide directed numbers

-3 -3

Two representations of the same calculation

2 x -3 = -6

Negative, Negative calculation

-2 x -3

This is the negative of 2 x -3

The act of making counters into their negative is turning them over

-2 x -3 = 6

Divisions are the inverse operations

Add directed numbers

= −1
= 1
Representations

2 + - 4 = -2

Zero pair
(-1 + 1 = 0)

Two "−1" left
= -2

8 + -3 = 5

Partitioning

8 + -3 = 5

5 + 3 + - 3 = 5

Partition the value to create a zero pair calculation

Generalisation

+ − = −

Multiples

4 4 4 4 4

Bar models can represent by something is a multiple. Eg. 20 is a multiple of 4

Lowest Common Multiples

LCM of 9 and 12

The first time their multiples match

LCM = 36

9 9, 18, 27, 36, 45, 54

12 12, 24, 36, 48, 60

9 18 27 36 45
12 24 36 48

Factors

1 2 3 6 9 18

Start at 1 and write the factor pair opposite.

Check every number.

When you get a repeat - you know you have all factors.

Square numbers have an **ODD number** of factors

Highest Common Factor

1 is a common factor of all numbers

16: 1, 2, 4, 8, 16

24: 1, 2, 3, 4, 6, 8, 12, 24

HCF=8

Addition/ Subtraction laws for indices

$3^5 \times 3^2 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3) \rightarrow 3^7$

The base number is all the same so the terms can be simplified

Addition law for indices

$a^m \times a^n = a^{m+n}$

$3^5 \div 3^2 \rightarrow 3^3$

$\frac{3 \times 3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} \rightarrow \frac{3^3}{3^0} \rightarrow \frac{3^3}{1}$

Subtraction law for indices

$a^m \div a^n = a^{m-n}$

Multiply expressions with indices

$4b \times 3a \equiv 4 \times b \times 3 \times a \equiv 4 \times 3 \times b \times a \equiv 12ab$

$5t \times 9t \equiv 5 \times t \times 9 \times t \equiv 5 \times 9 \times t \times t \equiv 45t^2$

$2b^4 \times 3b^2 \equiv 2 \times b \times b \times b \times b \times 3 \times b \times b \equiv 2 \times 3 \times b \times b \times b \times b \times b \times b \equiv 6b^6$

There are often misconceptions with this calculation but break down the powers

Divide expressions with indices

$\frac{24}{36} \rightarrow \frac{2 \times 2 \times 2 \times 3}{2 \times 3 \times 2 \times 3} \rightarrow \frac{2}{3}$

$\frac{5a^3b^2}{15ab^6} \rightarrow \frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b} \rightarrow \frac{a^2}{3b^4}$

Cross cancelling factors shows cancels the expression

$\frac{23a^7y^2}{5db^6}$ This expression cannot be divided (cancelled down) because there are no common factors or similar terms

Special indices laws

Anything to the power 0 = 1

Anything to the power 1 = itself

Model

$7^0 = 1$

$7^1 = 7$

Product law

$(4^2)^6 = 4^{12}$

$(a^8)^4 = a^{32}$

Multiply the indices, keep the base the same

Standard form with numbers > 1

Any number between 1 and less than 10 — $A \times 10^n$ — Any integer

Example

3.2×10^4

$= 3.2 \times 10 \times 10 \times 10 \times 10$

$= 32000$

Non-example

0.8×10^4

5.3×10^{07}

Negative powers of 10

0.001	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
$1 \times \frac{1}{1000}$	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
1×10^{-3}	0	0	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Positive powers of 10

1 billion – 1 000 000 000

$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Order numbers in standard form

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2} 2.4×10^2 3.3×10^0 1.3×10^{-1}

0.064 240 1 0.13

Look at the power first will the number be > or < than 1

Use a place value grid to compare the numbers for ordering

Numbers between 0 and 1

0.054

5.4×10^{-2}

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Round to powers of 10 and 1 sig figure

R If the number is halfway between we 'round up'

5495 to the nearest 1000: 5000 | 6000

5475 to the nearest 100: 5400 | 5500

5475 to the nearest 10: 5470 | 5480

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00037 to 1 significant figure is 0.0004

Round to the first non-zero number

Round to 1 significant figure

370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00000037 to 1 significant figure is 0.0000004

Round to the first non zero number

Estimate the calculation

Round to 1 significant figure to estimate

$4.2 + 6.7 \approx 4 + 7 \approx 11$ This is an **overestimate** because the 6.7 was rounded up more

The equal sign changes to show it is an estimation

$21.4 \times 3.1 \approx 20 \times 3 \approx 60$ This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors

Estimation

Estimations are useful – especially when using fractions and decimals to check if your solution is possible

Most estimations round to 1 significant figure

Estimations are useful – especially when using fractions and decimals to check if your solution is possible

$210 + 899 < 1200$

This is true because even if both numbers were rounded up, they would reach 300 + 900

The correct estimation would be 200 + 900 = 1100

Year 8 - Mathematics . . .

Milestone Assessment 2

Speak Like a Mathematician

3D - Three dimensions to the shape (length, width and height)

Face - A flat surface on a solid object

Plan - A drawing of something when drawn from above (sometimes birds eye view)

Scale - The comparison of something drawn to its actual size.

Scale Factor - The multiple that increases/ decreases a shape in size

Commutative - Changing the order of the operations does not change the result

Square root - A square root of a number is a number when multiplied by itself gives the value (√)

Square - A term multiplied by itself

Ratio - A statement of how two numbers compare

Proportion - A statement that links two ratios

Part - A section of a whole

Equivalent - Of equal value

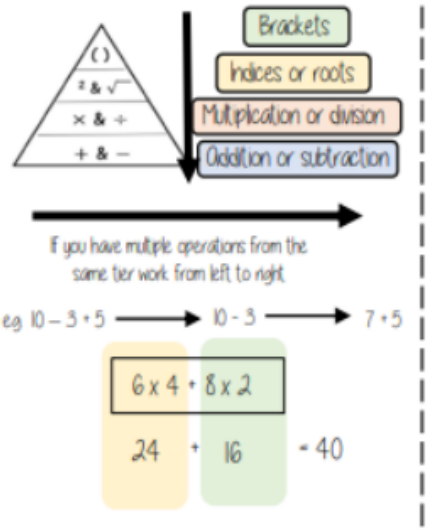
Simplify - To reduce to simplest form

Factors - Integers that multiply together to get the original value

Currency - The system of money used in a particular country

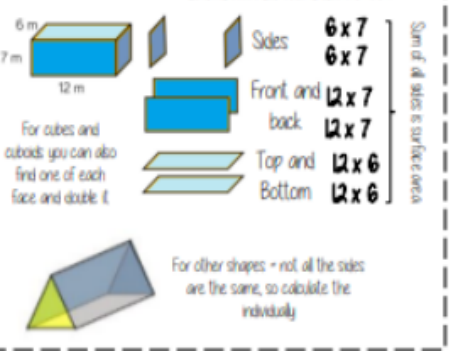
Conversion - The process of changing one variable to another

Order of operations

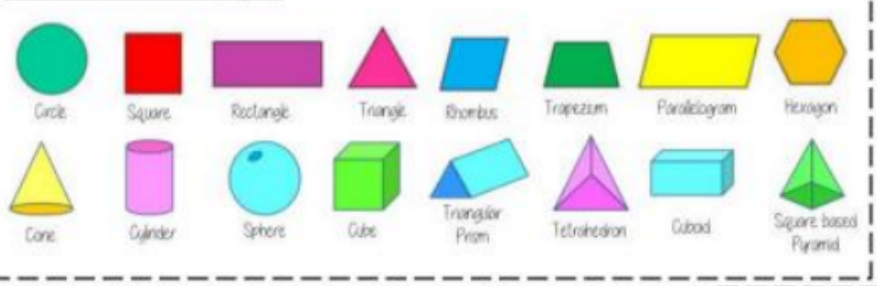


Surface area

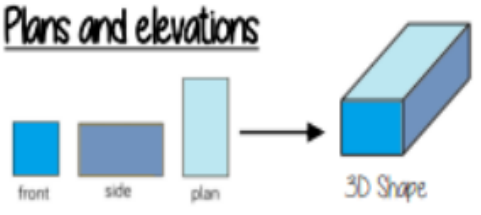
Sketching nets first helps you visualise all the sides that will form the overall surface area



Name 2D & 3D shapes

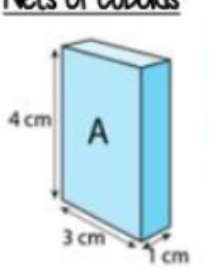


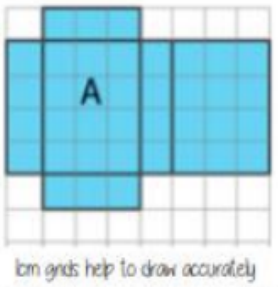
Plans and elevations



The direction you are considering the shape from determines the front and side views

Nets of cuboids

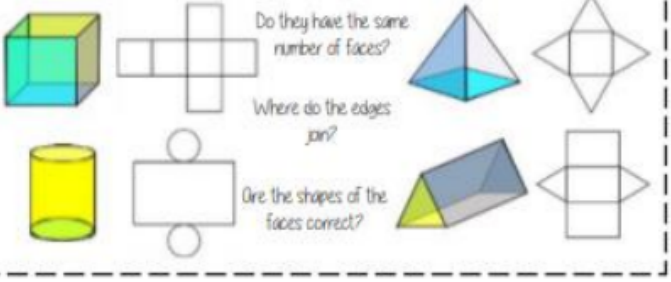




1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

Sketch and recognise nets



Representing a ratio

"For every 5 boys there are 3 girls"

5:3

This is the "whole" – boys and girls together

This represents the 5 boys

This represents the 3 girls

Double Number Line

This represents the 5 boys

This is the "whole" – boys and girls together

This represents the 3 girls

Order is Important

"For every dog there are 2 cats"

Dogs Cats

1:2

The ratio has to be written in the same order as the information is given.

e.g. 2:1 would represent 2 dogs for every 1 cat ✗

Simplifying a ratio

Cancel down the ratio to its lowest form

"For every 6 days of rain there are 4 days of sun"

6:4

÷ by 2

3:2

rain sun

Find the biggest common factor that goes into all parts of the ratio

For **6** and **4** the biggest factor (number that multiples into them is **2**)

"For every 3 days of rain there are 2 days of sun" – when this happens twice the ratio becomes 6:4

Ratio In (or n:1)

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1n

The question states that **this part has to be 1** unit. Therefore **Divide by 4**

4 : 20

1 : 5

This side has to be divided by 4 too – to keep in **proportion**

*The n part does not have to be an integer for this type of question

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4
Work out how much each person earns

Model the Question

James : Lucy

3 : 4

James

£350

Lucy

Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50

□ = one part = £50

Put back into the question

James : Lucy

(×50) 3 : 4 (×50)

£150 : £200

James = 3 x £50 = £150

Lucy = 4 x £50 = £200

£350

Finding a value given 1n (or n:1)

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1

□ = one part = 10 pens

Blue pens

Red pens

One unit = 10 pens

Put back into the question

Blue : Red

(×10) 5 : 1 (×10)

50 : 10

Blue pens = 5 x 10 = 50 pens

Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

Direct Proportion

As one variable changes the other changes at the same rate

This is a multiplicative change

4 cans of pop = £2.40

4 cans of pop = £2.40

2 cans of pop = £1.20 (×0.5)

4 cans of pop = £2.40

12 cans of pop = £7.20 (×3)

This multiplier is the same in the same way that this would be for ratio

Sometimes this is easiest if you work out how much one unit is worth first
e.g. **1 can of pop = £0.60**

Understand Scale Factor

The two rectangles are similar.

3 m, 8 m, 4.5 m, 7 m

3 x 1.5 = 4.5

This is a multiplicative change

Use **corresponding** sides to calculate a scale factor

Missing length
8 x 1.5 = 12m

Scale factor can also be calculated by

Bigger corresponding side
Smaller corresponding side

Small corresponding side ⟷ Big corresponding side
(× SF / ÷ SF)

Ratio between similar shapes

Angles in similar shapes do not change. e.g. if a triangle gets bigger the angles can not go above 180°

The two rectangles are similar.

3 m, 8 m, 4.5 m, 7 m

Corresponding sides

3m : 4.5m
1m : 1.5m (×3)

8m : 12m
1m : 1.5m (×8)

Note: Simplify to the same ratio

Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image : Real life
1cm : 30cm (×10)
10cm : 300cm (×10)

The car in real life is 210cm

Image : Real life
1cm : 30cm (×7)
7cm : 210cm (×7)

Conversion between currencies

£1 = 90 Rupees ← Currency is directly proportional

For every £1 I have 90 Rupees

£1 = 90 Rupees (×10)
£10 = 900 Rupees (×10)

Currency can be converted using a conversion graph

Rupees, 900, 10, Pounds

Convert 630 Rupees into Pounds

£1 = 90 Rupees (×7)
£7 = 630 Rupees (×7)

630 ÷ 90 = 7