# Probability

To know that the total probability sums to 1

To know that relative frequency is written in decimal form

To know that relative frequency is $\frac{number\ of\ outcomes}{Number\ of\ trials}$

To know that a sample space is the set of all possible outcomes of an experiment

To know that a Probability tree diagrams are a visual representation of a probability problem that involves a sequence of events

# ‘Solving equations and inequalities’

# The Knowledge for Progression:

- To know that an equation contains an equals symbol, variable and constant
- To know that an inequality contains an inequality symbol, variable and constant
- To know that equation/inequality are formed from expressions
- To know that solve means to find the value of the variable
- To know that solving always requires performing the inverse operations

| Key Word | Dual Coding | Definition |
|---|---|---|
| **Equation** | $4a + b - 12 = 32$ | Two expressions connected by an equal symbol |
| **Inequality** | $4a + b - 12 > 32$ | Two expressions connected by an inequality symbol |
| **Solve** | $\frac{x}{5} = 6$<br><br>$x = 30$ | Find the value of the variable |
| **Inverse** | $+ \longleftrightarrow -$<br><br>$\times \longleftrightarrow \div$<br><br>$a^2 \longleftrightarrow \sqrt{a}$ | Opposite operations that reverse the effect of the other operation |

## What do I need to be able to do?

**By the end of this unit you should be able to:**

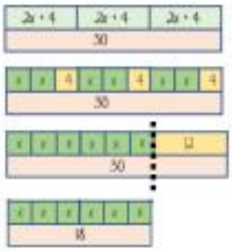
- Form and solve equations and inequalities
- Represent and interpret solutions on a number line as inequalities
- Draw straight line graphs and find solutions to equations
- Form and solve equations and inequalities with unknowns on both sides

## Keywords

**Solution:** a value we can put in place of a variable that makes the equation true
**Variable:** a symbol for a number we don't know yet
**Equation:** an equation says that two things are equal – it will have an equals sign =
**Expression:** numbers, symbols and operators grouped together to show the value of something
**Identity:** An equation where both sides have variables that cause the same answer includes ≡
**Linear:** an equation or function that is the equation of a straight line
**Intersection:** the point that two lines meet
**Inequality:** an inequality compares two values showing if one is greater than, less than or equal to another

## Solve equations (R)

$3(2x + 4) = 30$



$3(2x + 4) = 30$ — Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

$6x = 18$

$\div 6 \quad \div 6$

$x = 3$

Substitute to check your answer. This could be negative or a fraction or decimal

## Form and solve inequalities (R)

"Two more than treble my number is greater than 11"

**Form** $x \rightarrow \times 3 \rightarrow +2 \rightarrow 11$

$3x + 2 > 11$

**Solve** $x \leftarrow \div 3 \leftarrow -2 \leftarrow 11$

$x > 3$

## Solutions on a number line

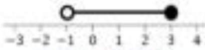
$x < 1$ and $x \le 1$: Both represent values less than 1. ($x \le 1$) Includes the value 1

$x > 1$ and $x \ge 1$: Both represent values more than 1. ($x \ge 1$) Includes the value 1

● Includes the value it sits above

○ Does NOT include the value it sits above

"Values less than or equal to 3 but also more than -1"



$-1 < x \le 3$

This includes the integer values 0,1,2,3

## Plotting straight line graphs (R)

$y = 3x - 1$ → 3 × the x coordinate then – 1

| $x$ | -3 | 0 | 3 |
|---|---|---|---|
| $y$ | -10 | -1 | 8 |

Draw a table to display the information

This represents a coordinate pair (-3, -10)

You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

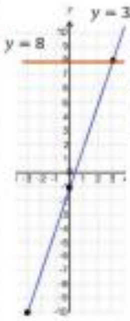
Remember to join the points to make a line.

## Find solutions graphically

For linear equations there is only one point the graph meets the x value

$x = 2$
$y = 4$

These two lines will cross at (2,4) because they are just x= and y= they are parallel to axes and meet in one place



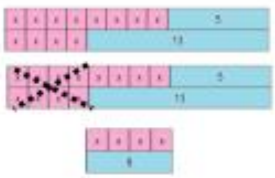
$3x - 1 = 8$

Remember equation of a line format is y = mx + c

The solution is the point the two lines meet **(3,8)**

## Equations: unknown on both sides (R)

$8x + 5 = 4x + 13$



$8x + 5 = 4x + 13$

$-4x \quad -4x$

$4x + 5 = 13$

$-5 \quad -5$

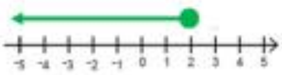
$4x = 8$

$\div 4 \quad \div 4$

$x = 2$

## Inequalities: unknown on both sides

$8x + 5 \le 4x + 13 \longrightarrow x \le 2$

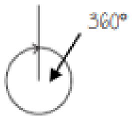


Any value 2 or less will satisfy the inequality

# Angle geometry

## Sum of angles at a point

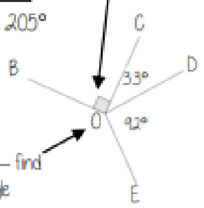
The sum of angles around a point is 360°



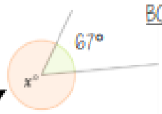

**Find angle BOE**

90° + 33° + 92° = 205°
360° - 205°
BOE = **155°**

Angle notation – 90°



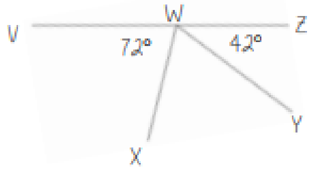

360° - 67°
= 293 °




Angle notation – find this missing angle

## Sum of angles on a straight line

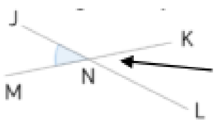
Adjacent angles that share a common point on a line add up to 180°




**Find angle XWY**

72° + 42° = 114°
180° - 114° = **66°**

## Vertically opposite angles

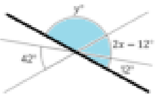



Angle JNM is vertically opposite to angle KNL

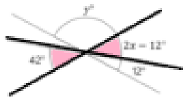
JNM = KNL

**Vertically opposite angles are the same**

Other angle rules still apply
Look for straight line sums and angles around a point



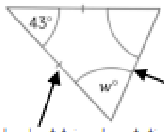

Form equations with information from diagrams




$2x - 12 = 42$
$2x = 54$
$x = 27°$

## Sum of angles in triangles

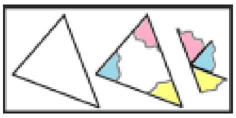
**Sum of interior angles in a triangle = 180°**




The two base angles will be the same size

Look at triangle notation
This indicates an isosceles triangle
∴ 180 - 43 = 137
137 ÷ 2 = 68.5°

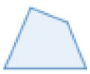
A triangle can only have ONE right angle



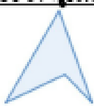
Have a go!
Tearing the corners from triangles forms a straight line which is therefore 180°

## Sum of angles in quadrilaterals

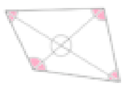
**Sum of interior angles in a quadrilateral = 360°**



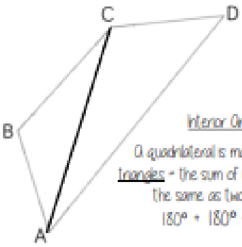
Convex Quadrilateral



Concave Quadrilateral



Interior angles are those that make up the perimeter (outline) of the shape

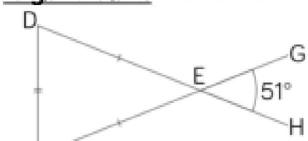



Interior Angles
A quadrilateral is made up of two triangles = the sum of interior angles is the same as two triangles
180° + 180° = 360°

## Angle Problems

Split up the problem into chunks and explain your reasoning at each point using angle notation

Keep working out clear and notes together

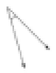



EDF = ____°

1. Angle DEF = 51° because it is a vertically opposite angle DEF = GEH
2. Triangle DEF is isosceles (triangle notation) ∴ EDF = EFD and the sum of interior angles is 180°
   180° - 51° = 129°    129° ÷ 2 = 64.5°
3. Angle EDF = 64.5°

## Basic angle rules and notation

The letter in the middle is the angle
The arc represents the part of the angle

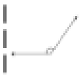


**Acute Angles**
0°< angle <90°

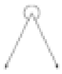
**Right Angles**
90°

Right angle notation


**Angle Notation:** three letters **ABC**
This is the angle at B = 113 °
**Line Notation:** two letters **EC**
The line that joins E to C



**Obtuse**
90°< angle <180°



**Reflex**
180°< angle <360°

**Straight Line**
180°

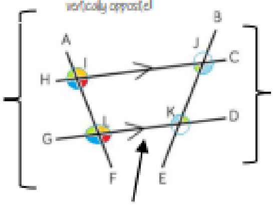
**Vertically opposite angles**
Equal
**Angles around a point**
360 °

## Parallel lines

Still remember to look for angles on straight lines, around a point and vertically opposite!

Lines AF and BE are **transversals** (lines that bisect the parallel lines)

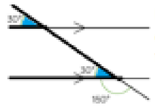
Corresponding angles often identified by their **"F shape"** in position




Alternate angles often identified by their **"Z shape"** in position

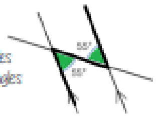
This notation identifies parallel lines

## Alternate/ Corresponding angles

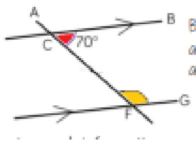



Because alternate angles are equal the highlighted angles are the same size

Because corresponding angles are equal the highlighted angles are the same size




## Co-interior angles



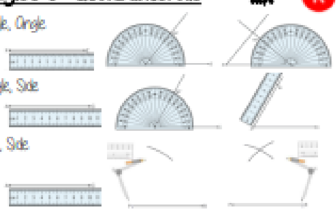

Because co-interior angles have a sum of 180° the highlighted angle is 110°

As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

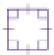
## Triangles & Quadrilaterals

Link to steps → R

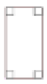
Side, Angle, Angle
Side, Angle, Side
Side, Side, Side



## Properties of Quadrilaterals

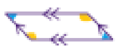


**Square**
All sides equal size
All angles 90°
Opposite sides are parallel

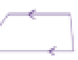


**Rectangle**
All angles 90°
Opposite sides are parallel

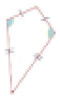
**Rhombus**
All sides equal size
Opposite angles are equal



**Parallelogram**
Opposite sides are parallel
Opposite angles are equal
Co-interior angles



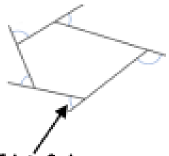
**Trapezium**
One pair of parallel lines



**Kite**
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

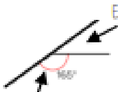
## Sum of exterior angles

**Exterior angles all add up to 360°**



**Exterior Angles**
Are the angle formed from the straight-line extension at the side of the shape

Using exterior angles




Exterior Angle
Interior Angle

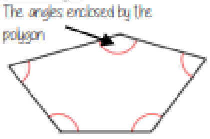
**Interior angle + Exterior angle = straight line = 180°**
Exterior angle = 180 – 165 = **15°**

**Number of sides = 360° ÷ exterior angle**
Number of sides = 360 ÷ 15 = 24 sides

## Sum of interior angles

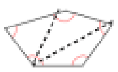
**(number of sides – 2) x 180**

**Interior Angles**
The angles enclosed by the polygon



This is an **irregular** polygon – the sides and angles are different sizes

Sum of the interior angles = (5 – 2) x 180

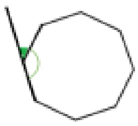


This shape can be made from three triangles
Each triangle has 180°

Sum of the interior angles = 3 x 180
= **540°**

Remember this is **all** of the interior angles added together

## Missing angles in regular polygons



Exterior angle = 360 ÷ 8 = **45°**

Interior angle = $\frac{(8-2) \times 180}{8} = \frac{6 \times 180}{8}$ = **135°**

**Exterior angles in regular polygons = 360° ÷ number of sides**

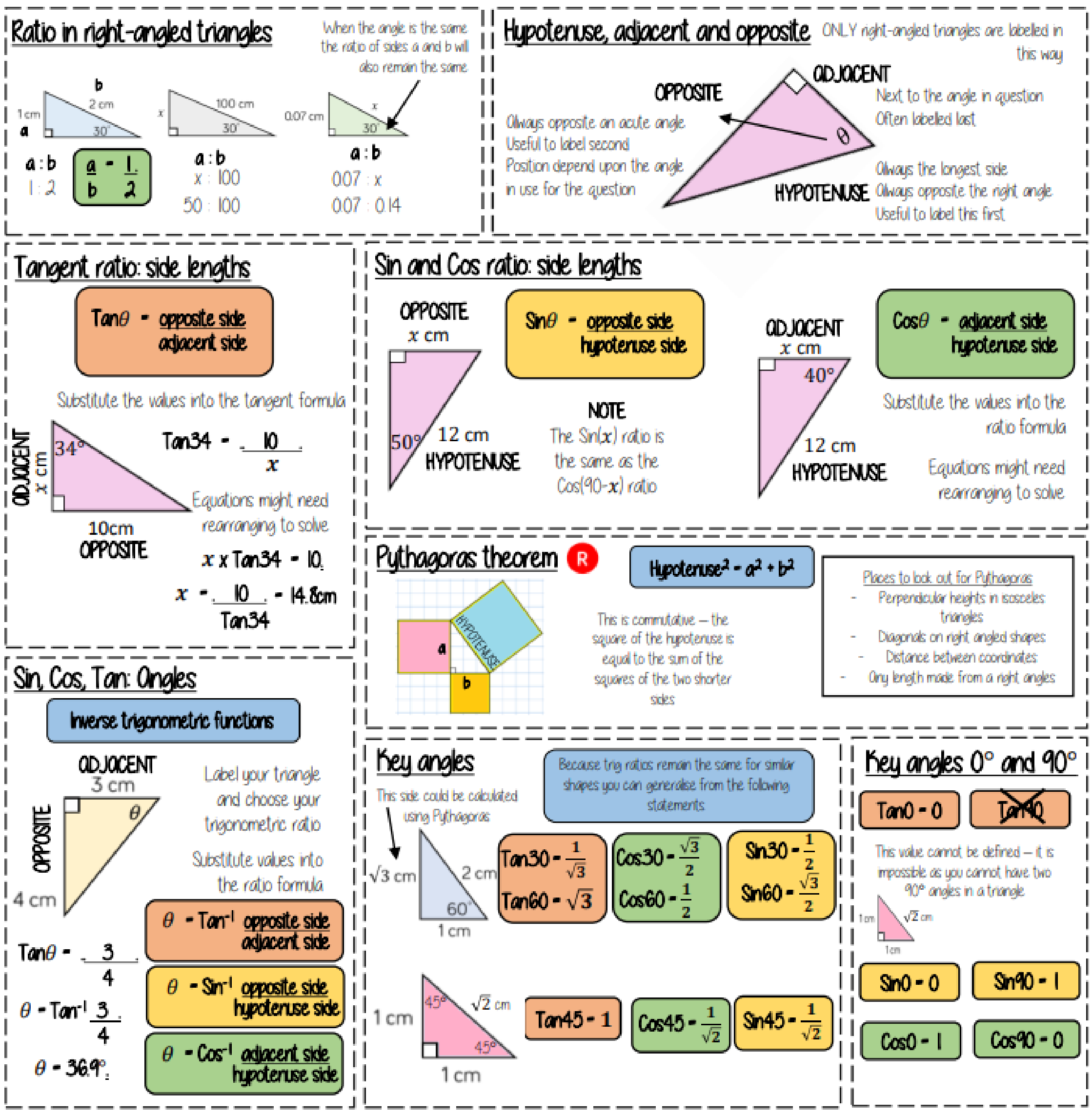
**Interior angles in regular polygons = $\frac{\text{(number of sides – 2) x 180}}{\text{number of sides}}$**

# ‘Trigonometry’

# The Knowledge for Progression:

- To know that trigonometry can only be applied to right-angled triangles where two sides and one angle are involved
- To know that you can label the sides hypotenuse, adjacent and opposite
- To know that the hypotenuse of a triangle is opposite the right-angle. This will always be the longest side of the triangle
- To know that the opposite side is opposite the angle involved (not the right-angle)
- To know that the adjacent side is next to the angle but is not the hypotenuse
- To know that

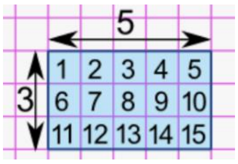
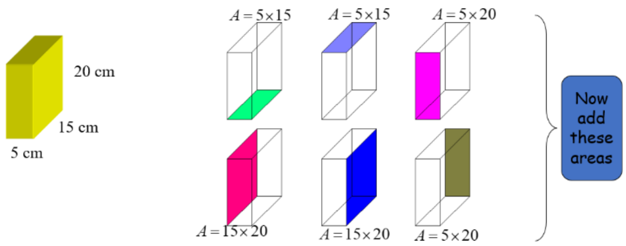
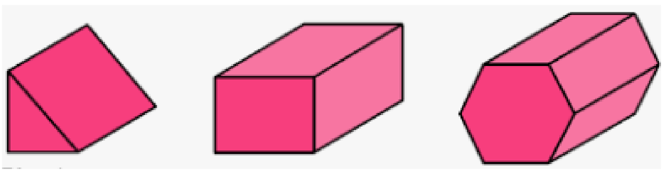
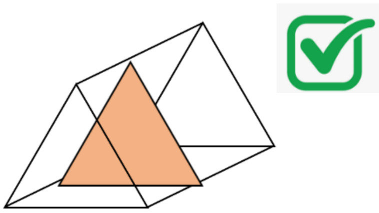
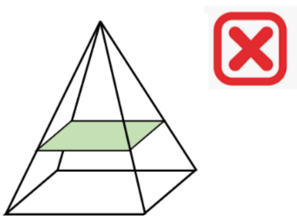
$$, \ Sin(angle) = \frac{Opposite}{Hyoptenuse} \quad Cos(angle) = \frac{Adjacent}{Hypotenuse} \quad Tan(angle) = \frac{Opposite}{Adjacent}$$



# ‘Surface area of prisms’

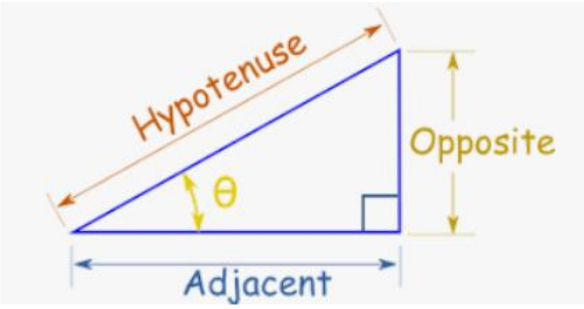
## The Knowledge for Progression:

- To know that surface area is the sum of the area of the faces of a 3D shape.
- To know that a face is a 2D side that makes up a 3D shape.
- To know that a prism is a 3D shape with a uniform cross section. The cross section is a polygon.
- To know that the uniform cross-section is the polygon that is runs throughout the prism.

| Key Word | Dual Coding | Definition |
|---|---|---|
| **Area** |  | The space inside a 2D shape |
| **Surface Area** |  | The total area of all the faces of a 3D shape added |
| **Prism** |  | A 3D shape with a uniform cross section. The cross section is a polygon |
| **Uniform cross-section** |   | The **same** face that runs through the length of a 3D shape. |

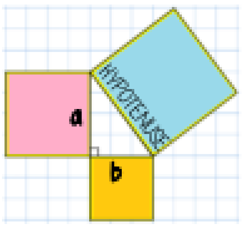
# ‘Pythagoras’

- To know that Pythagoras’ theorem can only be applied to right-angled triangles. It involves all three sides of the triangle.
- To know that the hypotenuse of a triangle is opposite the right-angle. This will always be the longest side of the triangle.
- To know $a^2 + b^2 = c^2$ where a and b are the shorter sides.

| Key Word | Dual Coding | Definition |
|---|---|---|
| **Hypotenuse** | | The longest length of a right-angled triangle. Always opposite the right-angle |
| **Opposite** |   | The length opposite the angle involved (not the right angle) |
| **Adjacent** | | The length next to the right angle, but not the hypotenuse |

## Pythagoras theorem

**Hypotenuse² = $a^2 + b^2$**



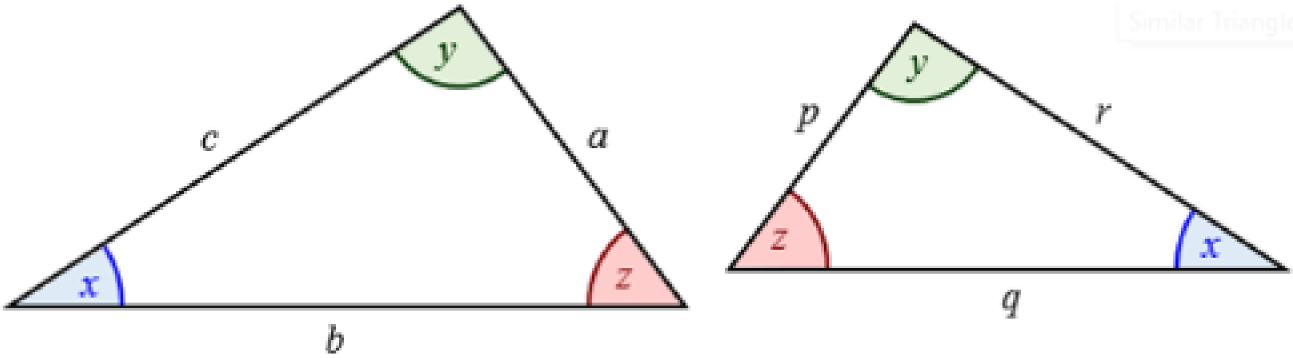

This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

# Similar Triangles

- Same shape, but not necessarily the same size.
- Corresponding angles are equal.
- Corresponding sides are in the same ratio.

$$\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$$




To test for similar triangles:

- AA – If 2 corresponding angles are equal.
- SSS – If 3 corresponding sides are in the same ratio.
- SAS – Ratio of 2 pairs of corresponding sides are equal and their included angles are equal.