Basic Algebra

Key vocabulary

Powers, Indices,
Reciprocal, Roots,
Brackets, Multiply out,
Expand, Simplify, Factor,
Factorise, Algebra,
Notation, Coefficient,
Expression, Equation,
Formula, Identity,
Inequality, Term,
Collecting like terms,

Picture perfect

Expanding brackets:
- Expanding means remove brackets
- Use the claw

e.g. $3(5a - 2) = (3 \times 5a) - (3 \times 2)$

$= 15a - 6$

Factorising expressions:
- The opposite (inverse) of expanding
- Answer will include brackets
- Look for common factors (numbers and algebra)
- Always choose the HCF

e.g. 1 $10a + 15 = 5(2a + 3)$

10 & 15 both in the 5 times table

$10a = 5 \times 2a$ $15 = 5 \times 3$

Assessment style question

Assessment style question: Circle the correct words to complete each sentence:

(a) $x^2 + 3x = 16$ is	an expression	an equation	a formula	an identity
(b) $x^2 + 3x = 16$ is	an expression	an equation	a formula	an identity
(c) $(x + 3)^2 \equiv x^2 + 6x + 9$ is	an expression	an equation	a formula	an identity

Always remember

Algebraic Notation

Algebra uses a shorthand system where variables (symbols) are represented by letters.

E.g. $a + a = 2a$

$2 \times b = 2b$

$c \times c = c^2$

$3d \times 2e = 6de$

Factorising Brackets

To factorise into one bracket, find the highest common factor for each term.

E.g. $4x + 10 = 2(2x + 5)$

Expanding Brackets

To expand one bracket:
make sure the term on the outside multiplies everything on the inside.

E.g. $4(2x - 3) = 8x - 12$

Term = 3a
Expression = 5x + 2
Equation = 5x − 3 = 47
Formula = C = πd
Identity= 10x + 4 ≡ 2(5x + 2)

Simplifying expressions (adding/subtracting)

- Can only simplify like terms.
- Be sure to include the sign before the term

e.g. $2a + 3b - a + 4b = a + 7b$

$2a - a = a$ $+ 3b + 4b = +7b$

Substitution and Formulae

Key vocabulary

Substitution

Formulae

Rearrange

Equation

BIDMAS

Picture perfect

I am think of a number. I subtract 5 from it and then divide the result by 4.

The answer is 7. What number did I think of to start with?

Form an equation then

$$\frac{x-5}{4} = 7$$

Solve the equation

$$\frac{x-5}{4} = 7$$

$$\times 4 \quad \times 4$$

$$x - 5 = 28$$

$$+5 \quad +5$$

$$x = 33$$

Solve this equation to find x:

$$6x = 13$$

$$\div 6 \qquad \div 6$$

$$x = \frac{13}{6}$$

Make x the subject of the formula:

$$a\,x = b$$

$$\div a \qquad \div a$$

$$x = \frac{b}{a}$$

Assessment style question

Work out the value of $d + (3e + f)^2$ when $d = 2$, $e = -3$ and $f = 1$.

Rearrange the formula $P = \frac{k}{j}$ to make k the subject of the formula.

Carl thinks of a number. He multiplies this by 3 then adds 23. Liz thinks of a number and she adds 3, then she multiplies this by 5. The number they chose and sums at the end are equal. What is the number they have picked?

Always remember

Substitution means putting numbers in place of letters to calculate the value of an **expression**.

For example, in the expression $2b^2c$, where $b = 4$ and $c = 3$, use the values of b and c to calculate the numerical value of the expression:

$$2b^2c = 2 \times b^2 \times c$$

Formulae are created for something that is calculated often.

For instance, plumbers often apply a call out charge plus an hourly rate to their customers. Writing a formula for the total cost of a job would be useful for a plumber so that they could quickly calculate costs for their customers more easily. Once a formula is written, the plumber would only need to input how long the job would take in hours, and come up with a total cost to quote very easily.

A rule or fact written with mathematical symbols.

It usually has:
• an equals sign (=)
• two or more variables (x, y, etc)
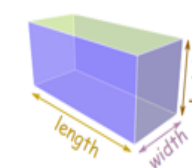
Example: The formula for the volume of a box is
V = l × w × h

Which has these variables:
• V stands for volume,
• l for length,
• w for width,
• h for height,

When l=10, w=5 and h=4, then V = 10 × 5 × 4 = 200

Sequences

Key vocabulary

Sequence
Term
Rule
Term to term rule
Nth term
Arithmetic sequence
Geometric sequence
Triangular numbers
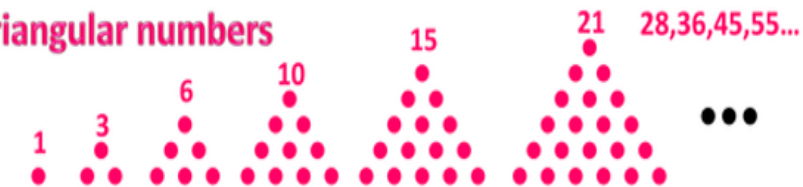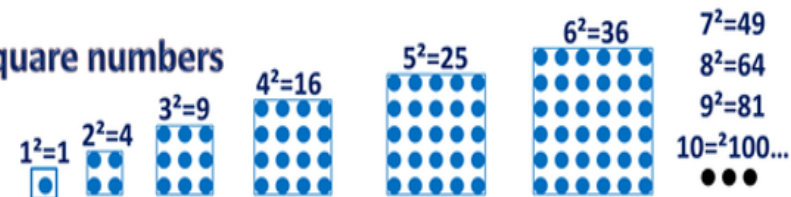Fibonacci sequence

Picture perfect

Sequence:

3, 5, 7, 9, ...

1st term
2nd term
3rd term
4th term

three dots means goes on forever (infinite)

("term", "element" or "member" mean the same thing)

ARITHMETIC SEQUENCE

-6, -1, 8, 15, 22

+7   +7   +7   +7

GEOMETRIC SEQUENCE

2, 4, 8, 16, 32

x2   x2   x2   x2

Assessment style question

The nth term of a sequence is $2n^2$

(i)   Find the 4th term of the sequence.

....

(ii) Is the number 400 a term of the sequence?

....

Give reasons for your answer.

....

....

The first five terms of an arithmetic sequence are

2   7   12   17   22

Write down, in terms of n, an expression for the nth term of this sequence.

Always remember

Fibonacci Sequence

$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987 \ldots$

Triangular numbers

1   3   6   10   15   21   28, 36, 45, 55...

Square numbers

$1^2=1$ $2^2=4$ $3^2=9$ $4^2=16$ $5^2=25$ $6^2=36$

$7^2=49$
$8^2=64$
$9^2=81$
$10^2=100\ldots$

Cube numbers

$1^3=1$ $2^3=8$ $3^3=27$

$4^3=64$
$5^3=125$
$6^3=216$
$7^3=343\ldots$

Solving equations

Key vocabulary

Substitution
Formulae
Rearrange
Equation
BIDMAS
Inverse
Coefficient

Picture perfect

Left Side $=$ Right Side

The equation is balanced

$$3(x + 1) = 2(x + 2)$$

$$3x + 3 = 2x + 4$$
$$-2x \qquad -2x$$

$$x + 3 = 4$$
$$-3 \quad -3$$

$$x = 1$$

Always remember

Solve this equation to find x:

$$6x = 13$$
$$\div 6 \qquad \div 6$$
$$x = \frac{13}{6}$$

Make x the subject of the formula:

$$a\,x = b$$
$$\div a \qquad \div a$$
$$x = \frac{b}{a}$$

I am think of a number. I subtract 5 from it and then divide the result by 4.
The answer is 7. What number did I think of to start with?

Form an equation then

$$\frac{x - 5}{4} = 7$$

Solve the equation

$$\frac{x - 5}{4} = 7$$
$$\qquad x4 \mid x4$$
$$x - 5 = 28$$
$$+5 \mid +5$$
$$x = 33$$

Assessment style question

Hannah is n years old.
Her aunt Emily is three times older than Hannah.
Emily is 48 years old.
   (a) Write down an equation for this information.
   (b) Solve your equation to find how old Hannah is.

Sam thinks of a number, n.
He multiplies his number by 7 and then adds 3 to the result.
His final answer is 45.
   (a) Write down an equation for this information.
   (b) Solve your equation to find the number, n.

The sum of each row is given.
Find a, b, c and d.

a	a	a	a	24
a	a	b	b	28
b	c	c	c	29
a	b	c	d	31

Inequalities
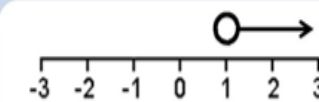
Key vocabulary

Less than
Less than or equal to
Greater than
Greater than or equal to
Represent on number line
Solve
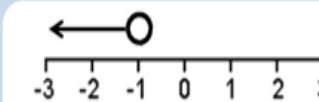
Picture perfect

Inequality Symbols

$<$ — Less Than

$>$ — Greater Than

$\leq$ — Less Than or Equal to

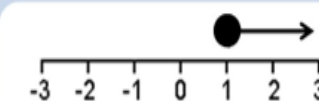$\geq$ — Greater Than or Equal to

Always remember

$x > 1$
x is any number greater than 1
Examples: 2, 3, 4, 5...

$x < -1$
x is any number less than -1
Examples: -2, -3, -4, -5...

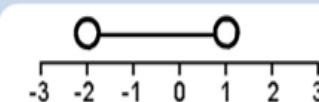$x \geq 1$
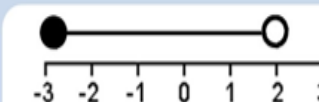x is any number greater than or equal to 1
Examples: 1, 2, 3, 4, 5...

$x \leq -1$
x is any number less than or equal to -1
Examples: -1, -2, -3, -4, -5...

$-2 < x < 1$
x is any number greater than -2 and less than 1
Examples: -1 and 0 only

$-3 < x \leq 2$
x is any number greater than or equal to -3 and less than 2
Examples: -3, -2, -1, 0 and 1 only

Assessment style question

Solve the inequality

$4x + 3 > 2(3x - 1)$

Answer _____ [3]

Choose the correct inequality that best describes each graph.

1)

a) $x \leq 1 + \frac{6x}{5}$ b) $1 + \frac{6x}{5} > x$

c) $1 + \frac{6x}{5} < x$ d) $x \geq 1 + \frac{6x}{5}$

2)

a) $15 < 3(4x - 7)$ b) $3(4x - 7) \leq 15$

c) $3(4x - 7) \geq 15$ d) $15 > 3(4x - 7)$

Coordinates & Linear Graphs

Key vocabulary

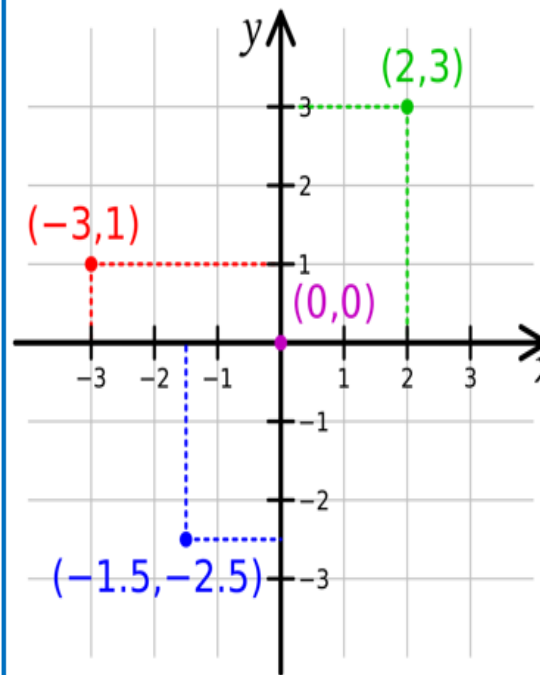**Coordinates** – a set of value that show an exact position on a coordinate grid

Linear equation – an equation, when plotted, makes a straight line

Gradient – the steepness of the line of a linear equation

y-intercept – where the linear equation cuts the y-axis

Substitution – when you replace an unknown for a given value

Picture perfect

(2,3)

(−3,1)

(0,0)

(−1.5,−2.5)

Reading and plotting coordinates

Coordinates are written in the form of (x, y). Remember the rule for reading and plotting coordinates is 'along the corridor and up or down the stairs'.

A = (-2, 4)   B = (4, 2)
C = (1, 2)   D = (-5, -3)
E = (4, -4)   F = (-4, -5)

Using a conversion graph

Conversion graphs can be used to convert between any 2 units which have a linear relationship.
Here, you can use the graph to convert between inches and centimetres

Drawing a graph to represent a linear equation

When you are asked to draw the line of a linear equation you follow these steps:
1. Complete a table of values by substituting in the value of x into the equation
2. Write the list of coordinates
3. Plot the coordinates
4. Draw a line through the all the coordinates

This means 2 multiplied the value of x, then add on 3

Assessment style question

Question 1:

(a) Plot the coordinates A (−4, 1), B (1, −2) and C (2, 1)
(b) ABCD is a kite.
(c) Plot D
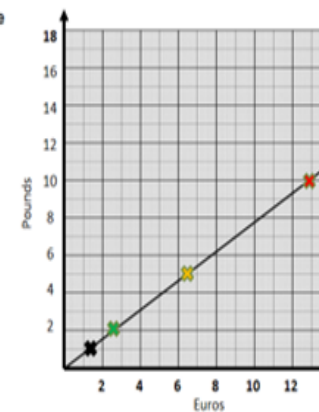(d) Write down the coordinates of the point D.

Question 2:

The distance between the points (−3, −4) and (q, 5) is 15.
Find the possible values of q.

Drawing a conversion graph

You can plot known conversions on a graph to help you to convert other unknown amounts.

Can you use the graph to convert 10 Euros into Pounds?

Current exchange rate
£1 = € 1.29
£2 = € 2.58
£5 = € 6.45
£ 10 = € 12.90

Example

Complete the table of values for the equation y = 2x + 3 for the values of x from 0 to 2

X	0	1	2
	2x0+3	2x1+3	2x2+3
y	=3	=5	=7

Coordinates are (0,3) (1,5) and (2,7)