

Place Value and Negatives

Key vocabulary

Place Value: The value given to a digit by its place in a number

Digit: Single numbers, 0 – 9, used to write a whole number

Integers: Whole numbers

Decimals: Show parts of a whole number

Negative: Less than zero

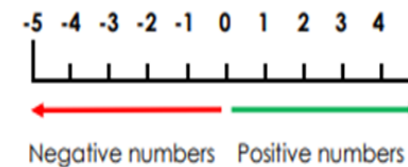
Picture perfect

Decimal Place Value Chart

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths	Millionths
M	Hth	TTh	Th	H	T	O	t	h	th	tth	hth	m

Positive numbers are any numbers more than zero e.g. 1, 2, 3, 4, 5.

Negative numbers are any numbers less than zero e.g. -1, -2, -3, -4, -5.



Assessment style question

Here are four digits

6 3 9 7

(a) Put one digit in each box to make the smallest possible total.

+

Negatives: Tristan is taking part in a maths competition. Each correct answer is worth 5 points and each incorrect answer is worth -3. If Tristan chooses not to answer a question, it is worth 0 points. There are 10 questions in total.

(a) What would Tristan's final score be if he answered 5 correctly, 4 incorrectly and left 1 blank?

(b) Can Tristan finish with -10 points? Explain your answer.

When we put numbers in order, we need to compare the value of their digits.

2,123,518 2,123,736 2,122,845

First, look at the millions digits in each number. Each number has the same digit in the millions place so you then keep comparing digits of the same place value until you find ones that are different. The thousands digits are different so that tells us that 2,122,845 is the smallest number because it has a 2 in the thousands place. Looking at the hundreds digits, we can see that 2,123,518 is the next smallest.

2,122,845 2,123,518 2,123,736

Smallest

Always Remember

Example : $-8 + 12$

When adding and subtracting with negative numbers, you should use a number line. Start at the first number given in the sum (here, it's -8).



Then, think about whether you are adding or subtracting your number. If you're adding the number needs to get bigger, so you move to the right each time. If you're subtracting, the number must need to be smaller, so you move left. We need to add 12 in our example, so add 8 to get back to zero, then add on in 1's until you get to 12...



The number you end up on is your answer! $-8 + 12 = 4$

Rules for **multiplying** with negative numbers:-

Positive x Positive = Positive
Negative x Negative = Positive
Positive x Negative = Negative
Negative x Positive = Negative

Examples

$5 \times 4 = 20$
 $-3 \times -2 = 6$
 $10 \times -7 = -70$
 $-8 \times 9 = -72$

✓ Where the signs are the same, the product of the numbers is positive!

✓ Where the signs are different, the product of the numbers is negative!

Rules for **dividing** with negative numbers:-

Positive ÷ Positive = Positive
Negative ÷ Negative = Positive
Positive ÷ Negative = Negative
Negative ÷ Positive = Negative

Examples

$20 \div 4 = 5$
 $-6 \div -2 = 3$
 $70 \div -7 = -10$
 $-72 \div 9 = -8$

✓ Where the signs are the same, the quotient of the numbers is positive!

✓ Where the signs are different, the quotient of the numbers is negative!

+ - x ÷ BIDMAS

Key vocabulary

Sum – The result of adding 2 or more numbers together.

Product – The result of multiplying.

Indices – Made up of a base and a power, the power tells us how many times to multiply the base by itself.

Assessment style question

Joey thinks the answer to $16 + 4 \times 2$ is 40.
Albert thinks the answer to $16 + 4 \times 2$ is 24.

Who is correct?
Explain your answer.

Kyle is organising a charity concert at school.
The concert is sold out.
The hall holds 28 rows of 16 seats.
Each person will pay £6.

How much money will Kyle raise for charity?

Put brackets in the following statements to make them true

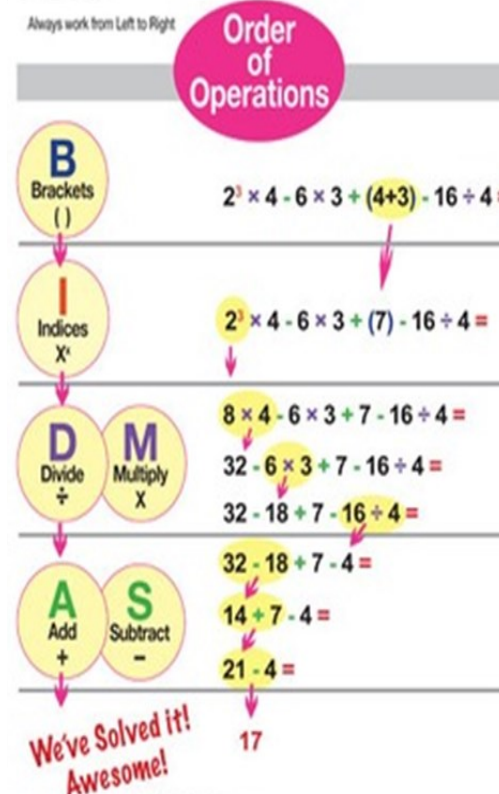
(a) $6 \times 7 + 3 - 8 = 52$

(b) $4 + 3 \times 7 - 1 = 42$

Poppy has £7.04 credit on her mobile phone.
It costs 8p to send a text message.

How many text messages can Poppy send?

Picture perfect



Always remember



ADDITION

add
plus
and
total

+

increase
more
sum
together

MULTIPLICATION

multiply
times
product
multiplied by

×

groups of
lots of
doubled
times tables

SUBTRACTION

take away
minus
less
reduce
remain

-

take from
fewer
take
difference
how many more

DIVISION

divided by
share
divide
divide into

÷

divisible by
group
each
share equally

Factors and multiples

Key vocabulary

Multiples – The times tables of a specific number.

LCM – Lowest common multiple.

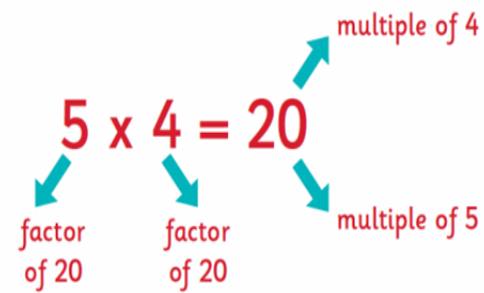
Factor – An integer which divides into another fully with no remainder.

HCF – Highest common factor.

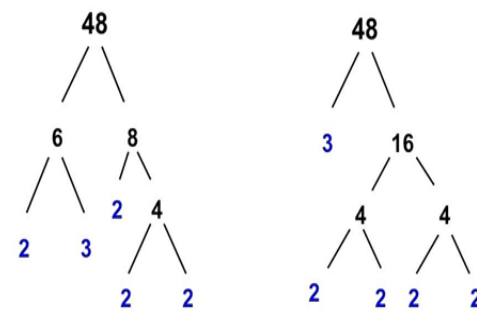
Prime number – An integer with only 2 factors.

Prime factorisation – Writing a number as a product of its prime factors.

Picture perfect



Prime factorisation

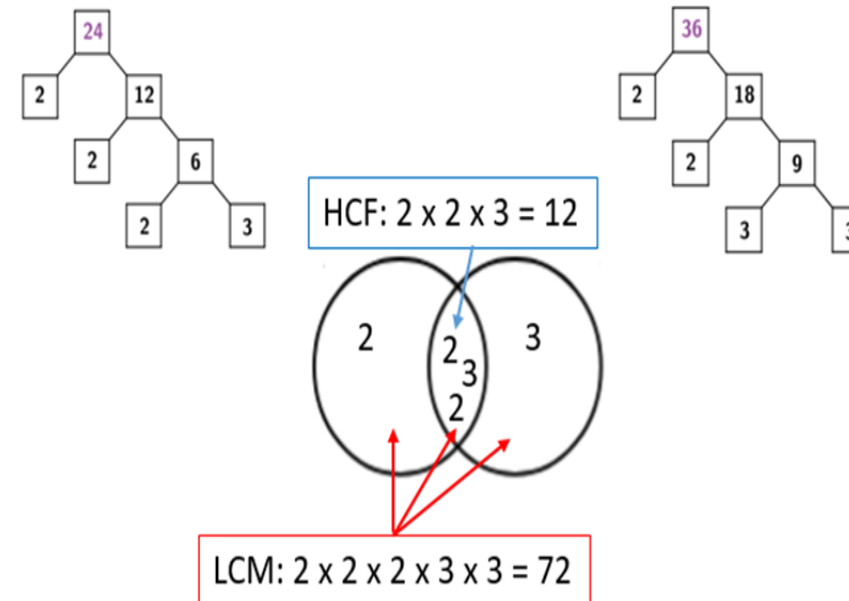


$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Always remember

HCF and LCM

Find the HCF and LCM of 24 and 36



Assessment style question

Mary is organising a charity hot dog sale.
There are 18 bread rolls in each packet.
There are 15 hot dogs in each packet.
Mary buys exactly the same number of bread rolls as hot dogs.

What is the smallest number of each packet that Mary can buy?

The Highest Common Factor (HCF) of two numbers is 6.
The Lowest Common Multiple (LCM) of the same numbers is 60.

What are the two numbers?

2 is the only even prime number.

1 is NOT a prime number because it only has 1 factor.

Powers and Roots

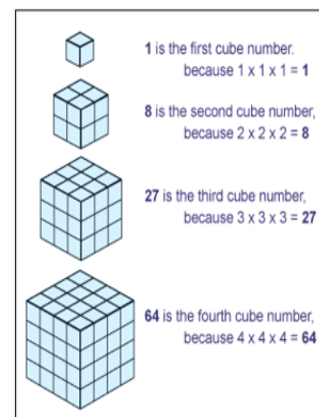
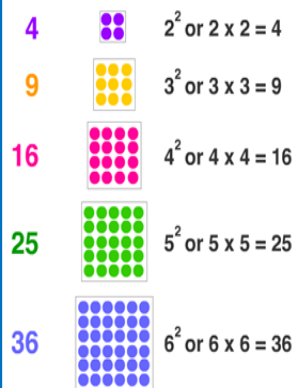
Key vocabulary

Square number – Answer to a number multiplied by itself,
e.g. $2 \times 2 = 4$

Cube number – Answer to a number multiplied by itself 3 times
e.g. $2 \times 2 \times 2 = 8$

Indices – Made up of a base and a power, the power tells us how many times to multiply the base by itself.

Picture perfect



Always remember

$$\begin{array}{llllll} \sqrt{0} = 0 & \sqrt{16} = 4 & \sqrt{64} = 8 & \sqrt[3]{0} = 0 & \sqrt[3]{64} = 4 & \sqrt[3]{512} = 8 \\ \sqrt{1} = 1 & \sqrt{25} = 5 & \sqrt{81} = 9 & \sqrt[3]{1} = 1 & \sqrt[3]{125} = 5 & \sqrt[3]{729} = 9 \\ \sqrt{4} = 2 & \sqrt{36} = 6 & \sqrt{100} = 10 & \sqrt[3]{8} = 2 & \sqrt[3]{216} = 6 & \sqrt[3]{1000} = 10 \\ \sqrt{9} = 3 & \sqrt{49} = 7 & & \sqrt[3]{27} = 3 & \sqrt[3]{343} = 7 & \end{array}$$

Negative powers change numbers to fractions

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

Power $\frac{1}{2}$ is the same as square root. Power $\frac{1}{3}$ is the same as cube root

$$\begin{array}{l} 10^1 = 10 \\ 10^2 = 10 \times 10 = 100 \\ 10^3 = 10 \times 10 \times 10 = 1,000 \\ 10^4 = 10 \times 10 \times 10 \times 10 = 10,000 \\ 10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100,000 \\ 10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000 \end{array}$$

Numbers in standard form

number between 1 and 10 power of 10

$$1.2 \times 10^3$$

Page 8

Assessment style question

The population of the United Kingdom in 1950 was 5.06×10^7

The population of the United Kingdom in 2015 was 6.47×10^7

Work out how many more people live in the United Kingdom in 2015 than 1950.
Give your answer as an ordinary number.

Question 3: Work out

(a) $64^{\frac{1}{3}} \times 2^3$

Arrange in order from smallest to largest.

$$\frac{1}{50} \quad 5^{-2} \quad \frac{3}{10} \quad 2^{-3}$$

Fractions

Key vocabulary

Fraction - A quantity which is not a whole number.

Decimal - A decimal number is often used to mean a number that uses a decimal point followed by digits that show a value smaller than one.

Percentage - Amount out of one hundred.

Improper fraction - The numerator is larger than the denominator.

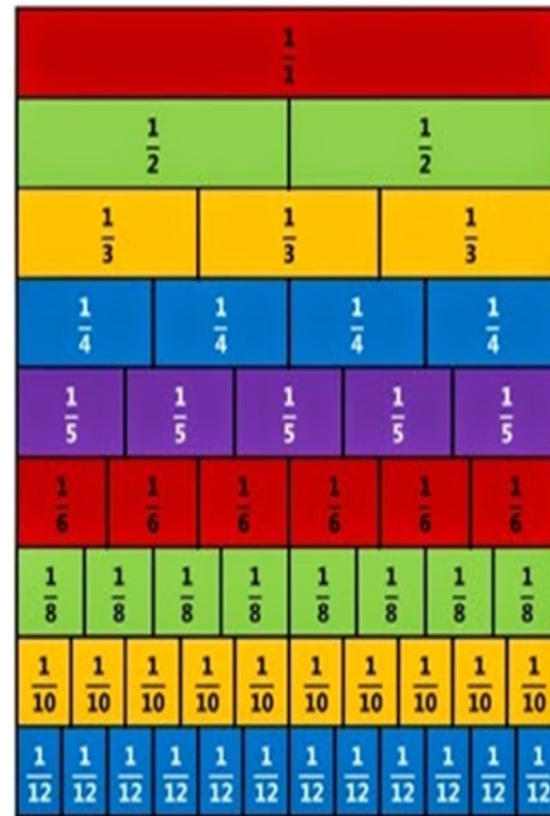
Mixed Number - A whole number and a fraction together.

Equivalent - When 2 amounts are equal they are equivalent

Numerator - The top number of a fraction.

Denominator - The bottom number of a fraction.

Picture perfect



Always remember

Decimals, Percentages and Fractions			
Fraction	Percentage	Decimal	
1 whole	100%	1	
$\frac{1}{2}$	50%	0.5	
$\frac{1}{3}$	33.3%	0.33	
$\frac{1}{4}$	25%	0.25	
$\frac{1}{5}$	20%	0.2	
$\frac{1}{6}$	16.7%	0.167	
$\frac{1}{8}$	12.5%	0.125	
$\frac{1}{10}$	10%	0.1	
$\frac{1}{12}$	8.3%	0.083	

Fractions Decimals Percentages

Converting Fraction to Decimal or Decimal to Percentage



Converting Percentage to Decimal or Decimal to Fraction



$\frac{1}{2}$ → Numerator
2 → Denominator

When converting to fractions: find the denominator and then cancel down if necessary

'Per cent' (%) means 'out of 100'. 'Of' means 'multiply'

Multiplying Fractions



Multiply the numerators
Multiply the denominators

$$\frac{5}{6} \times \frac{1}{11} = \frac{5}{66}$$

Dividing Fractions



$$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$$

Adding and subtracting simple fractions

We can use **equivalent** fractions to add fractions that do not have the same **denominator**.

For example:

$$\frac{3}{4} + \frac{1}{8}$$

We need to change $\frac{3}{4}$ into an equivalent fraction with a denominator of 8.

$$\frac{3}{4} = \frac{6}{8}$$

Now we have:

$$\frac{6}{8} + \frac{1}{8} = \frac{7}{8}$$

Assessment style question

Shown is a rectangle.
Find the value of x

$$\text{Area} = 20\text{cm}^2 \quad 2\frac{1}{6}\text{ cm}$$

A wall measures $3\frac{3}{4}\text{ m}$ by $4\frac{1}{3}\text{ m}$

Each can of paint cover 2.5m^2 and costs £5.50

Work out the cost of painting the wall.



Dave and Tom are discussing fractions.
Is either man correct?

$\frac{4}{5}$ is equivalent to $\frac{16}{20}$

Dave

$\frac{4}{5}$ is equivalent to $\frac{20}{24}$

Tom

Percentages

Key vocabulary

Fraction - A quantity which is not a whole number.

Decimal - A decimal number is often used to mean a number that uses a decimal point followed by digits that show a value smaller than one.

Percentage - Amount out of one hundred.

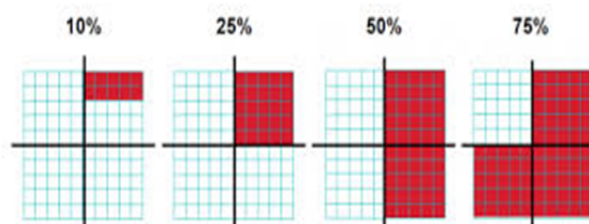
Increase - To make bigger.

Decrease - To make smaller.

Depreciate - Decrease in value over time.

Multipliers - a quantity by which a given number is to be multiplied.

Picture perfect



Reverse percentages

John pays £60 for a bag after getting 20% discount. How much did it originally cost?

Remember: Original price is always equal to 100%

$$\text{Sale price} = 100\% - 20\% = 80\%$$



Assessment style question

A primary school has 212 students.
50% of the students are boys.
How many of the students are boys?

A fish tank, that is full of water, has sprung a leak.
12% of the water is lost every hour.
What percentage of the water is lost after three hours?

A cereal bar weighs 24g.
The cereal bar contains 3.8g of protein.
Work out what percentage of the cereal bar is protein.

When a tennis ball is dropped, it bounces and then rises.
The ball rises to 80% of the height from which it is dropped.
The ball is dropped from a height of 4 metres.

- Calculate the height of the rise after the first bounce.
- Calculate the height of the rise after the second bounce.

The ball carries on bouncing, each time rising to 80% of the last rise.

- For how many bounces does the ball rise to a height greater than 10cm?

Dorothy organises a charity raffle.
She sells 800 tickets for £2 each.
4% of the tickets win a prize that costs £20.
65% of the profit goes to Charity A and the rest goes to Charity B.
How much money does Dorothy raise for Charity B?

Always remember

Decimals, Percentages and Fractions			
Fraction	Percentage	Decimal	
1 whole	100%	1	
$\frac{1}{2}$	50%	0.5	
$\frac{1}{3}$	33.3%	0.33	
$\frac{1}{4}$	25%	0.25	
$\frac{1}{5}$	20%	0.2	
$\frac{1}{6}$	16.7%	0.167	
$\frac{1}{8}$	12.5%	0.125	
$\frac{1}{10}$	10%	0.1	
$\frac{1}{12}$	8.3%	0.083	

Fractions Decimals Percentages

Converting Fraction to Decimal or Decimal to Percentage



Converting Percentage to Decimal or Decimal to Fraction



on a calculator

$$39\% \text{ of } 82$$

$$0.39 \times 82$$

Change to a decimal and multiply

increasing

Increase £60 by 12%

$$12\% \text{ of } 60 = 0.12 \times 60 = £7.20$$

$$\text{New amount} = £60 + £7.20 = £67.20$$

ADD

fraction to %

$$\frac{15}{20} = \frac{75}{100} = 75\%$$

$$15 \div 20 \times 100 = 75\%$$

Percentages

decreasing

decrease £60 by 12%

$$12\% \text{ of } 60 = 0.12 \times 60 = £7.20$$

$$\text{New amount} = £60 - £7.20 = £52.80$$

SUBTRACT

without a calculator

- 50% - half
- 25% - half and half
- 75% - 50% + 25%
- 10% - divide by 10
- 5% - half 10%
- 20% - double 10%

Simple interest = amount \times multiplier \times time

Compound interest = amount \times multiplier^{time}

Rounding and Estimating

Key vocabulary

Rounding - to alter a number making it less accurate but easier to use in calculations.

Estimation - A rough calculation using a mathematical method.

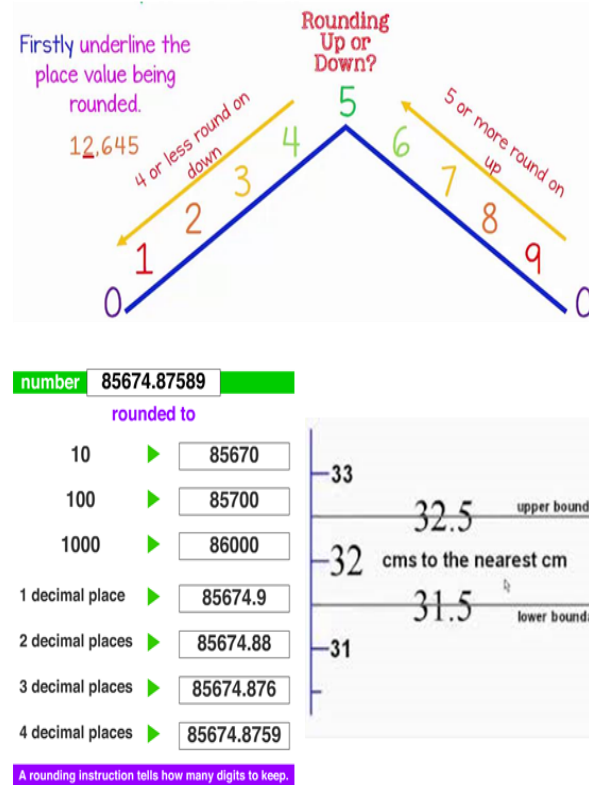
Decimal place - Place value after the decimal point.

Integer - A whole number.

Significant figures (S.F. of Sig Fig) - Digits that carry meaning.

Bounds - Upper and lower values of rounded numbers.

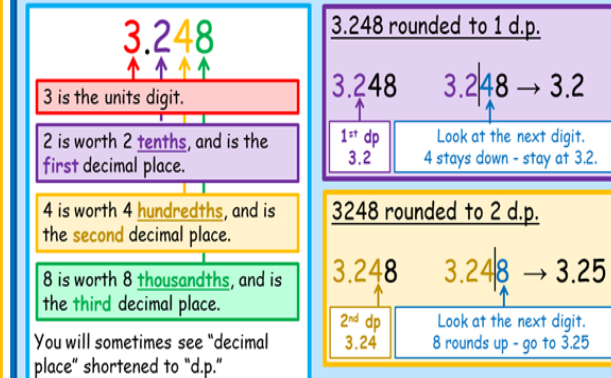
Picture perfect



Always remember

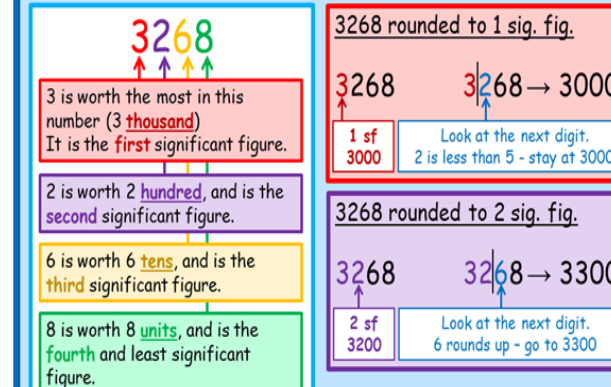
Rounding to decimal places

Rounding to decimal places is exactly like rounding whole numbers - you just have more numbers (and therefore greater accuracy).



Significant figures

If something is **significant**, it is big or important. The **most significant** thing is the biggest or most important thing.



Assessment style question

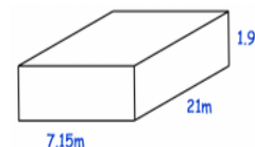
Nicole has rounded a number to one decimal place. Her answer is 9.2

Write down 10 different possible numbers that she could have rounded.

In an election 43.8% of people voted for a candidate. Round this figure to one significant figure

Andrew fills the swimming pool with water at a constant rate of 2.1 litres per second.

Given $1\text{m}^3 = 1000$ litres, estimate how long it takes to fill the pool.



A roll of wallpaper cost £7.85. Richard buys 29 rolls of wallpaper. Work out an estimate for the total cost.

We estimate by rounding all values to 1 significant figure before completing the calculation.