

## Algebra introduction

### Key vocabulary

Variable  
Terms  
Like terms  
Solve  
Simplify  
Expression  
Equation  
Identity  
Expand  
Factorise  
Co-efficient  
Inequality

### Picture perfect

Here is  $x + 2$ :

$$\begin{array}{|c|c|c|} \hline x & 1 & 1 \\ \hline \end{array}$$

$3(x + 2)$  means 3 lots of  $x + 2$  and would look like this:

$$\begin{array}{|c|c|c|} \hline x & 1 & 1 \\ \hline x & 1 & 1 \\ \hline x & 1 & 1 \\ \hline \end{array}$$

Altogether this is  $3x + 6$ .  
Algebraically, we would write:  
 $3(x + 2) = 3x + 6$ .

We have multiplied each term inside the bracket by 3.

### Always remember

#### Algebraic Notation

Algebra uses a shorthand system where variables (symbols) are represented by letters.

E.g.  $a + a = 2a$

$$2 \times b = 2b$$

$$c \times c = c^2$$

$$3d \times 2e = 6de$$

#### Expanding brackets:

- Expanding means remove brackets
- Use the claw

e.g.  $3(5a - 2) = (3 \times 5a) - (3 \times 2)$   
 $= 15a - 6$

Term -  $3a$

Expression -  $5x + 2$

Equation -  $5x - 3 = 47$

Formula -  $C = \pi d$

Identity -  $10x + 4 \equiv 2(5x + 2)$

#### Simplifying expressions (adding/subtracting)

- Can only simplify like terms.
- Be sure to include the sign before the term

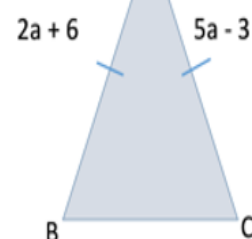
e.g.  $2a + 3b - a + 4b = a + 7b$

$$\begin{array}{c} (2a - a) = a \\ (+3b + 4b) = +7b \end{array}$$

### Assessment style question

ABC is an isosceles triangle.

- a) Find the value of  $a$ .  
b) Find the length AB.



$$\bullet 5(x + 2)$$

Factorise

$$w^2 - 5w$$

$$\bullet x(x - 6)$$

Factorise completely

$$24x^2 + 20x$$

$$\bullet 3x(2 - 9x)$$

To solve two step equations, you want to isolate your variable by itself. To do so, you must use your knowledge of inverse operations.

$$\begin{array}{r} 8b + 5 = 29 \\ -5 \quad -5 \\ \hline 8b = 24 \\ \div 8 \quad \div 8 \\ \hline b = 3 \end{array}$$

Step 1: Subtract 5 from both sides.  
Step 2: Divide both sides by 8.

You must do the inverse

$$\begin{array}{r} -18x + 30 = -150 \\ -30 \quad -30 \\ \hline -18x = -180 \\ \div -18 \quad \div -18 \\ \hline x = 10 \end{array}$$

Inverse operations				
operation	+	=	×	÷
inverse	=	+	÷	×

#### Factorising expressions:

- The opposite (inverse) of expanding
- Answer will include brackets
- Look for common factors (numbers and algebra)
- Always choose the HCF

e.g. 1  $10a + 15 = 5(2a + 3)$

10 & 15 both in the 5 times table

$$10a = 5 \times 2a \quad 15 = 5 \times 3$$

# Formulae

## Key vocabulary

Substitution  
Formulae  
Rearrange  
Equation  
BIDMAS

## Picture perfect

I am think of a **number**. I **subtract 5** from it and then **divide the result by 4**.

The **answer is 7**. What number did I think of to start with?

Form an equation then Solve the equation

$$\frac{x-5}{4} = 7$$

$$\frac{x-5}{4} = 7$$

$$\times 4 \quad \times 4$$

$$x-5 = 28$$

$$+5 \quad +5$$

$$x = 33$$

Solve this equation to find x:

$$6x = 13$$

$$\div 6$$

$$\div 6$$

$$x = \frac{13}{6}$$

Make x the subject of the formula:

$$ax = b$$

$$\div a$$

$$\div a$$

$$x = \frac{b}{a}$$

## Assessment style question

Work out the value of  $d + (3e + f)^2$  when  $d = 2$ ,  $e = -3$  and  $f = 1$ .

Rearrange the formula  $P = \frac{k}{j}$  to make  $k$  the subject of the formula.

Carl thinks of a number. He multiplies this by 3 then adds 23.  
Liz thinks of a number and she adds 3, then she multiplies this by 5.  
The number they chose and sums at the end are equal. What is the number they have picked?

## Always remember

**Substitution** means putting numbers in place of letters to calculate the value of an **expression**.

For example, in the expression  $2b^2c$ , where  $b = 4$  and  $c = 3$ , use the values of  $b$  and  $c$  to calculate the numerical value of the expression:

$$2b^2c = 2 \times b^2 \times c$$

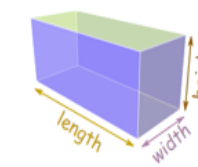
Formulae are created for something that is calculated often.

For instance, plumbers often apply a call out charge plus an hourly rate to their customers. Writing a formula for the total cost of a job would be useful for a plumber so that they could quickly calculate costs for their customers more easily. Once a formula is written, the plumber would only need to input how long the job would take in hours, and come up with a total cost to quote very easily.

A rule or fact written with mathematical symbols.

It usually has:

- an equals sign (=)
- two or more variables (x, y, etc)



Example: The formula for the volume of a box is  $V = l \times w \times h$

Which has these variables:

- V stands for volume,
- l for length,
- w for width,
- h for height,

When  $l=10$ ,  $w=5$  and  $h=4$ , then  $V = 10 \times 5 \times 4 = 200$

## Solving equations


### Key vocabulary

Substitution  
Formulae  
Rearrange  
Equation  
BIDMAS  
Inverse  
Coefficient

### Picture perfect

Left Side = Right Side

The equation is balanced



$$3(x + 1) = 2(x + 2)$$

$$\begin{array}{r} 3x + 3 = 2x + 4 \\ -2x \quad -2x \\ \hline x + 3 = 4 \\ -3 \quad -3 \\ \hline x = 1 \end{array}$$

### Always remember

Solve this equation to find x:

$$\begin{array}{r} 6x = 13 \\ \div 6 \quad \div 6 \\ \hline x = \frac{13}{6} \end{array}$$

Make x the subject of the formula:

$$\begin{array}{r} ax = b \\ \div a \quad \div a \\ \hline x = \frac{b}{a} \end{array}$$

I am think of a **number**. I **subtract 5** from it and then **divide the result by 4**.

The **answer is 7**. What number did I think of to start with?

Form an equation then Solve the equation

$$\begin{array}{r} x - 5 = 7 \\ \underline{+5} \\ x = 12 \end{array}$$

$$\begin{array}{r} x - 5 = 7 \\ \underline{+5} \\ x = 12 \end{array}$$

### Assessment style question

Hannah is n years old.  
Her aunt Emily is three times older than Hannah.  
Emily is 48 years old.

- Write down an equation for this information.
- Solve your equation to find how old Hannah is.



The sum of each row is given.  
Find a, b, c and d.

a	a	a	a	24
a	a	b	b	28
b	c	c	c	29
a	b	c	d	31

Sam thinks of a number, n.  
He multiplies his number by 7 and then adds 3 to the result.  
His final answer is 45.

- Write down an equation for this information.
- Solve your equation to find the number, n.

# Sequences

## Key vocabulary

Sequence

Term

Rule

Term to term rule

Nth term

Arithmetic sequence

Geometric sequence

Triangular numbers

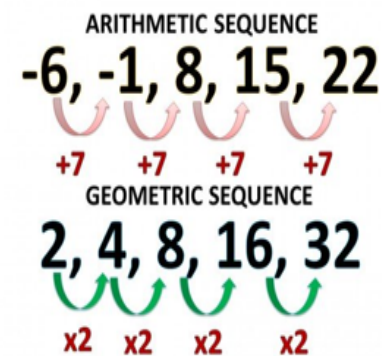
Fibonacci sequence

## Picture perfect

Sequence:



("term", "element" or "member" mean the same thing)

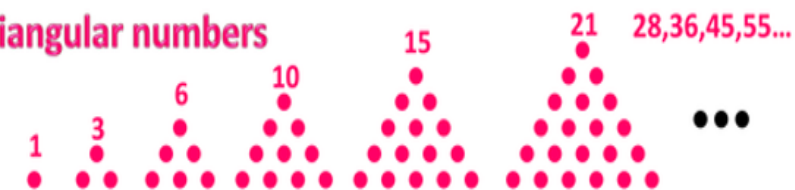


## Always remember

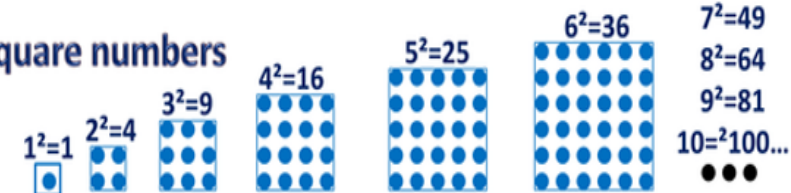
# Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987 ...

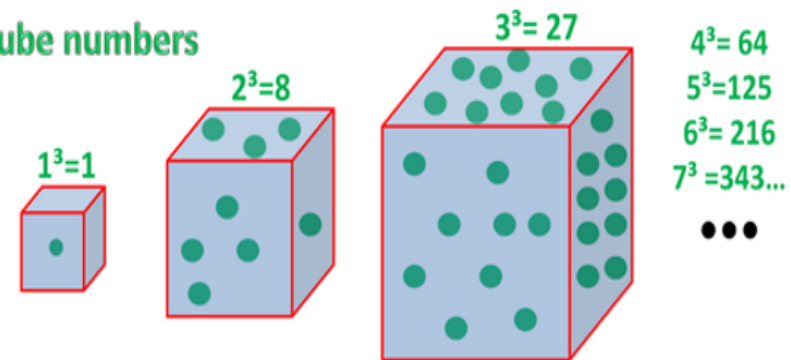
## Triangular numbers



## Square numbers



## Cube numbers



## Assessment style question

The  $n$ th term of a sequence is  $2n^2$

(i) Find the 4th term of the sequence.

(ii) Is the number 400 a term of the sequence?

Give reasons for your answer.

The first five terms of an arithmetic sequence are

2 7 12 17 22

Write down, in terms of  $n$ , an expression for the  $n$ th term of this sequence.



# Linear graphs

## Key vocabulary

Co-ordinates  
Quadrant  
Origin  
Equation  
Horizontal  
Vertical  
Parallel  
Intersect  
Gradient  
Plot  
Sketch  
Linear

## Picture perfect

All straight-line graphs have an equation that is written that is written like this:

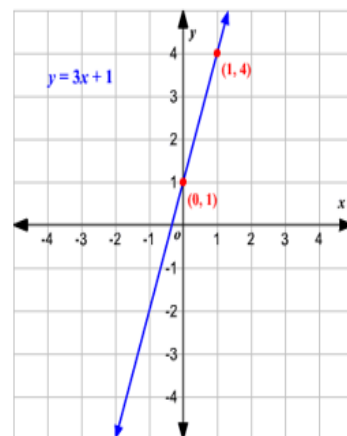
$$y = mx + c$$

Drawing Graphs  
Step 1 – Draw a table up  
Step 2 – Plot co-ordinates on axes  
Step 3 – Join the points

M is the gradient of the line, the 'steepness' of it.

X is the x-coordinate we need to plot the graph

C is called the y-intercept. It's the point where the line crosses the y-axis.



## Always remember

A line sloping upwards from left to right has a <b>positive</b> gradient.		m is positive
A line sloping downwards from left to right has a <b>negative</b> gradient.		m is negative
Parallel lines have the <b>same</b> gradient.		$m_1 = m_2$
A <b>horizontal</b> line has a gradient of 0.		$m = 0$
The gradient of a <b>vertical</b> line is undefined.		m is undefined

Coordinates are given in the form  $(x, y)$ . The first number is the position in the  $x$ -direction (horizontal) and the second is the position in the  $y$ -direction (vertical).

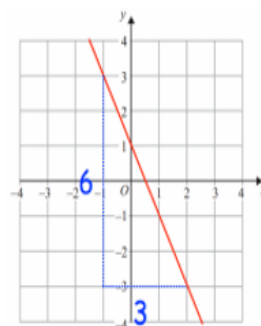
$$\text{Gradient} = \frac{\text{change up}}{\text{change right}} \text{ or } \frac{\text{change in } y}{\text{change in } x}$$

## Assessment style question

(a) Draw  $y = x + 1$  and  $y = 2x - 1$  on the same set of axes.

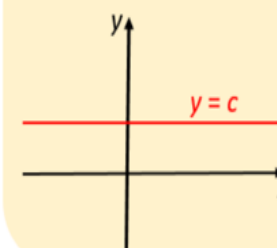
(b) Where do the two graphs intersect?

Alisha says that the gradient of the line is 2. Explain her mistake.



## Graph Horizontal and Vertical Lines

A **horizontal** line has an equation of the form  $y = c$ . It has a slope of 0.



A **vertical** line has an equation of the form  $x = a$ . It has a an undefined slope.

