Probability
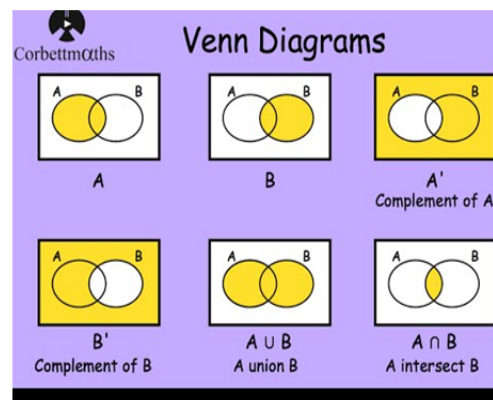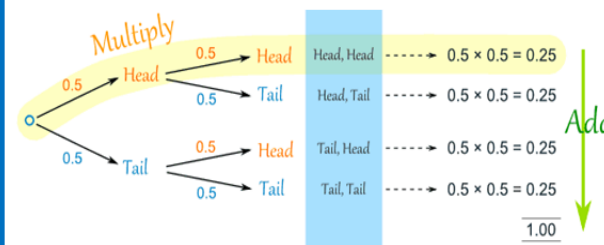
Key vocabulary

Probability
Random
Likely
Impossible
Certain
Relative frequency
Probability scale
Unbiased
Theoretical probability
Sample space
Two way table
Tree diagram
Venn diagram
Dependant
Independent
Conditional probability

Picture perfect

Venn Diagrams

A
B
A'
Complement of A

B'
Complement of B
A ∪ B
A union B
A ∩ B
A intersect B

Assessment style question

19 Wendy goes to a fun fair.
She has one go at Hoopla.
She has one go on the Coconut shy.

The probability that she wins at Hoopla is 0.4
The probability that she wins on the Coconut shy is 0.3

(a) Complete the probability tree diagram.

(2)

(b) Work out the probability that Wendy wins at Hoopla and also wins on the Coconut shy.

Always remember

Types of events

Mutually exclusive

Events that cannot happen at the same time
Rolling a die → $P(1\ and\ 6)$

All probabilities from the event will **sum** to make **1**

Independent events

Events where the outcome of one **doesn't** affect the outcomes of the others
Picking a counter out of a bag, replacing it and repeating.
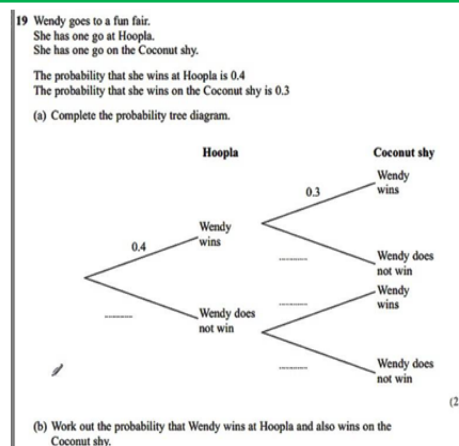
Dependent events

Events where the outcome of one **does** affect the outcomes of the others
Picking a counter out of a bag, **not** replacing it and repeating.

Calculating expected outcomes

$$P(event) \times number\ of\ trials$$

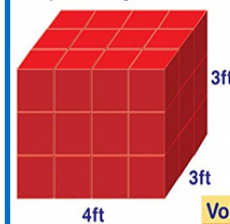# Maths - Foundation

Volume

Key vocabulary

Volume
Cross section
Length
Width
Height
Pi (π)
Circumference
Diameter
Radius
Units^3

Picture perfect
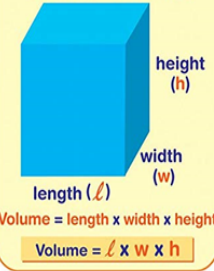
Volume is the space inside a 3D shape.

Volume

Volume is the number of cubic units that fill up a solid figure.

$V = \ell \times w \times h$
$V = 4 \times 3 \times 3$
$V = 36$

Volume = 36 cubic feet

Volume formula

height (h)
width (w)
length (ℓ)

Volume = length x width x height

Volume = ℓ x w x h

Always remember

Volume units include mm^3, cm^3, m^3 etc.

Volume of a Cylinder

$V = Bh$

Volume, Base Area, Height

$B = \pi r^2$

Base Area, Radius
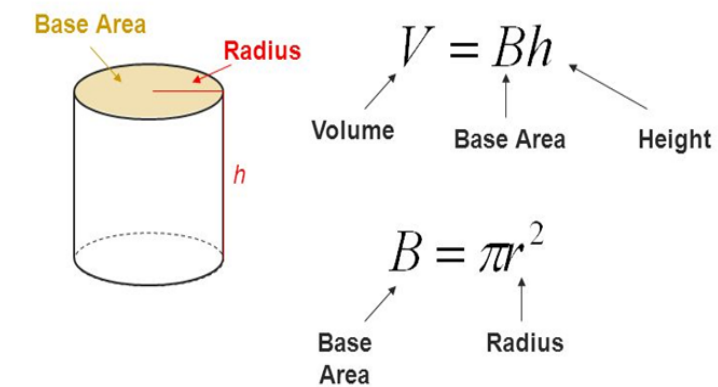
Volume of triangular prism = area of cross-section × length

$$= \frac{1}{2} \times b \times h \times l$$

Volume of Sphere $= \frac{4}{3}\pi r^3$

Volume of Cone $= \frac{1}{3}\pi r^2 h$

Page 5

Assessment style question

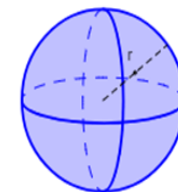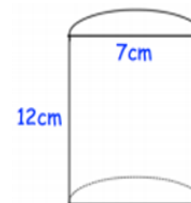Calculate the volume of this shape.

7cm
12cm
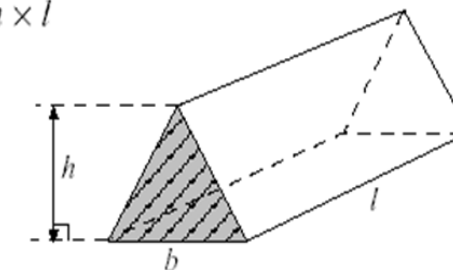
The cube on the TV show "The Cube" is a cube with each side measuring 4m. Work out the volume of the cube.

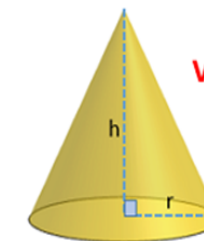The cuboid and the triangular prism have the same volume. Find x.

8cm
20cm
5cm
3.2cm
5cm
x
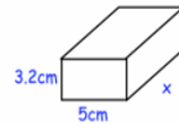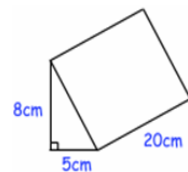
A solid sphere fits perfectly inside of a cube box of side length 10cm. What percentage of the box is empty?

Quadratic equations

Key vocabulary

Quadratic equations
Binomial
Expand
Simplify
Factorise
Difference of two squares
Rearrange formulae

Picture perfect

Factorising
$= 8n^2 + 20n$
$4n(2n + 5)$

Highest Common Factor

Assessment style question

Make **p** the subject of the following equation:

$$m = 2p - 5$$

A) $p = \frac{m}{2} + 5$ C) $p = \frac{m+5}{2}$

B) $p = 2m - 5$ D) $p = \frac{m-5}{2}$

3. (i) Factorise $x^2 - 13x + 36$

    (ii) Hence, or otherwise, solve the equation $x^2 - 13x + 36 = 0$

Always remember

Solving:
• Factorising
• Formula
• Completing the square
• Drawing a graph

Quadratic Equations
$ax^2 + bx + c$

Completing the square:
$x^2 + 4x - 3 = 0$
$(x + 2)^2 - 4 - 3 = 0$
$(x + 2)^2 - 7 = 0$
$x + 2 = \pm\sqrt{7}$
$x = \pm\sqrt{7} - 2$

half of 4x
subtract 2^2

Factorising:
easy...
$x^2 + 7x + 12 = 0$
$(x + 3)(x + 4) = 0$
$x = ^-3$ or $x = ^-4$

brackets

... more difficult!

$3x^2 - 5x + 2$
$3x^2 - 3x - 2x + 2$
$3x(x - 1) - 2(x - 1)$
$(3x - 2)(x - 1)$

multiply
$2x3$
$1x6$
6
1x6

Difference of Two Squares:
$x^2 - 16$
$(x - 4)(x + 4)$

x squared subtract 4 squared

The formula:
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Graphs:

Parabola – u shaped graph

draw lines to find solutions

Scatter graphs

Key vocabulary

Scatter graph
Plot
Positive correlation
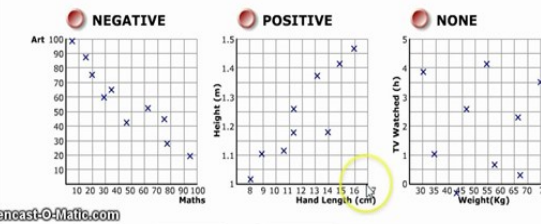Negative correlation
No correlation
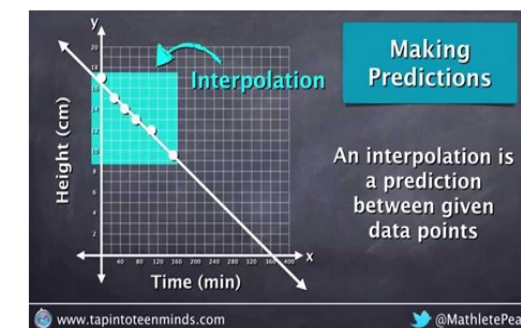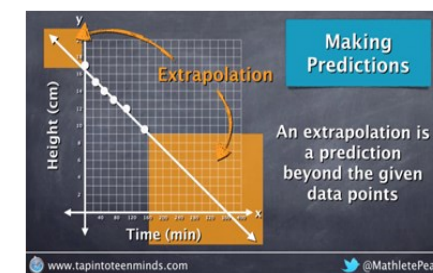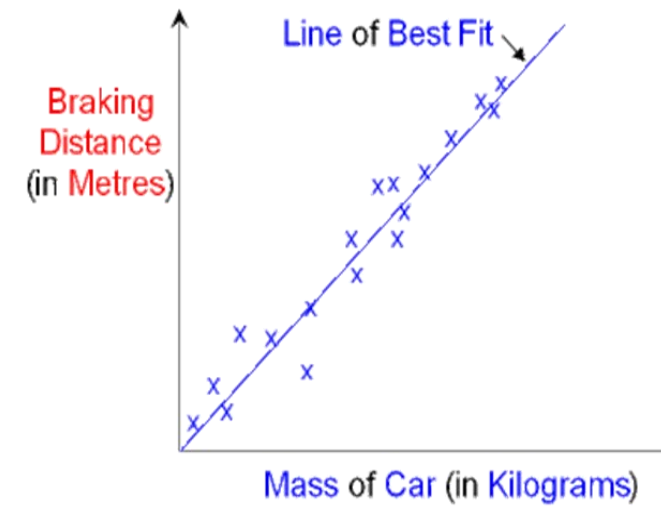Line of best fit
Interpolate
Extrapolate

Picture perfect

Summary

A Scatter Graph helps us to determine whether there is a link between two sets of data.
If a graph has a 'direction' we can fit a Line of Best Fit to it.
If there is a link we call it a Correlation.
These three graphs show example of;

NEGATIVE   POSITIVE   NONE

Always remember

Line of Best Fit
Braking Distance (in Metres)
Mass of Car (in Kilograms)

Making Predictions

An extrapolation is a prediction beyond the given data points

Making Predictions

An interpolation is a prediction between given data points

Assessment style question

5. Make a scatter plot with a proper title (at the bottom) with the data and answer the question using hypothesis test.

The yearly data have been published showing the number of releases for each of the commercial movie studios and the gross receipts for those studios thus far. Based on these data, can it be concluded that there is relationship between the number of releases and the gross receipts?

No. of release x	311	290	206	32	33	15	9	11	18
Gross receipts y (millions $)	3744	2062	1571	1664	214	301	258	134	155

Plot the following scatter graph and estimate a line of best fit.

Height, cm	157	160	148	160	177	156	166	170
Weight, kg	53	60	44	53	54	60	54	70

Inequalities

Key vocabulary
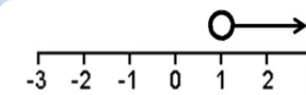
Less than
Less than or equal to
Greater than
Greater than or equal to
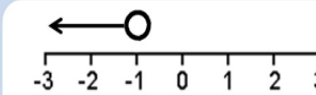Represent on number line
Solve

Picture perfect

Inequality Symbols

$<$ — Less Than

$>$ — Greater Than

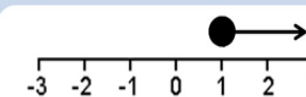$\leq$ — Less Than or Equal to

$\geq$ — Greater Than or Equal to

Always remember

x > 1
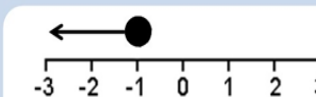
x is any number greater than 1

Examples: 2, 3, 4, 5...

x < - 1

x is any number less than -1
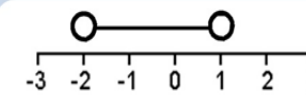
Examples: -2, -3, -4, -5...

x ≥ 1

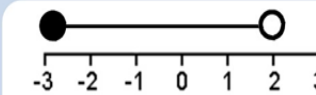x is any number greater than or equal to 1

Examples: 1, 2, 3, 4, 5...

x ≤ - 1

x is any number less than or equal to -1

Examples: -1, -2, -3, -4, -5...

- 2 < x < 1

x is any number greater than -2 and less than 1

Examples: -1 and 0 only

-3 < x ≤ 2

x is any number greater than or equal to -3 and less than 2

Examples: -3, -2, -1, 0 and 1 only

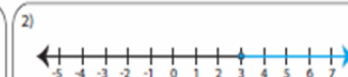### Assessment style question

Solve the inequality

$4x + 3 > 2(3x - 1)$

Answer _____ [3]

Choose the correct inequality that best describes each graph.

1)

a) $x \leq 1 + \frac{6x}{5}$ b) $1 + \frac{6x}{5} > x$

c) $1 + \frac{6x}{5} < x$ d) $x \geq 1 + \frac{6x}{5}$

2)

a) $15 < 3(4x - 7)$ b) $3(4x - 7) \leq 15$

c) $3(4x - 7) \geq 15$ d) $15 > 3(4x - 7)$
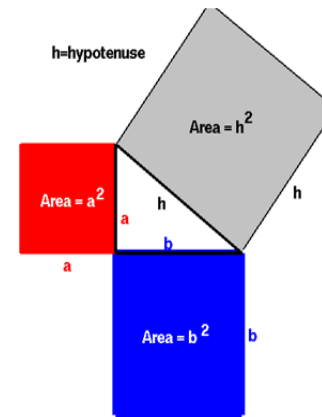
Pythagoras' theorem

Key vocabulary

Hypotenuse
Right angled triangle
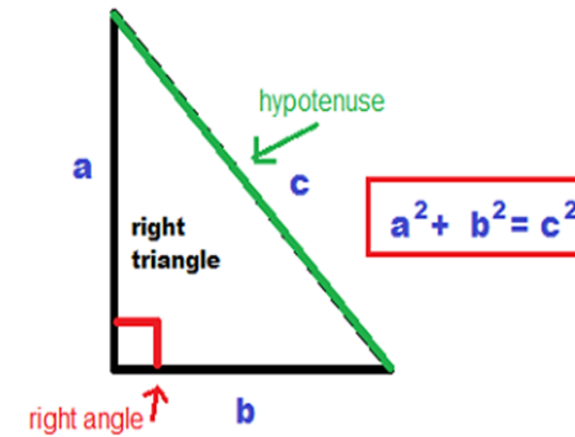Rearrange
Solve
Missing side
Square
Square root

Picture perfect

For any right triangle with sides a and b and hypotenuse h, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

h=hypotenuse

Area = h^2

Area = a^2

Area = b^2

$$h^2 = a^2 + b^2$$

Assessment style question

Find the length of the missing side of each triangle.
Give your answers to 2 d.p. where appropriate.

? cm   4 cm   3 cm

7 cm   9 cm   ? cm

5 cm   13 cm   ? cm

10 cm   5 cm   ? cm

Always remember

hypotenuse

a

c

right triangle

right angle

b

$$a^2 + b^2 = c^2$$

Find the missing length x

X is the hypotenuse c

$a^2 + b^2 = c^2$

$6^2 + 8^2 = x^2$

$36 + 64 = 100$

$x^2 = 100$

$x = 10$ cm

x   8 cm   6 cm

13 cm   5 cm   X

Find the missing length x

13 cm is the hypotenuse c

$a^2 + b^2 = c^2$

$x^2 + 5^2 = 13^2$

$x^2 + 25 = 169$

$x^2 = 144$

$x = 12$ cm
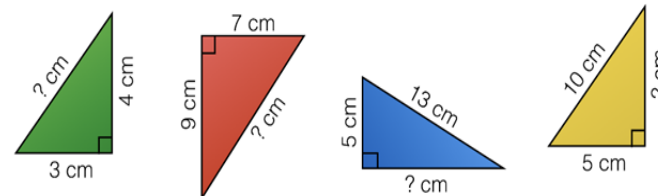
Simultaneous equations

Key vocabulary

Solve

Substitute

Eliminate

Variable

Linear

Quadratic

Simultaneous

Picture perfect

What are simultaneous equations?

$x + y = 5$

$2x + 3y = 12$

- If x=3 and y=2 <u>both</u> equations are true

- If you are asked to *solve simultaneous equations*, you are being asked to *find the values for x and y* that *fit* <u>both</u> the equations.

Always remember

This is when you have two equations with two unknowns which you can solve either algebraically or graphically.

Example 1
Solve the simultaneous equations:
$$3x + y = 7 \ (1)$$
$$5x + y = 5 \ (2)$$
Label the equations 1 and 2.
The coefficient of the y values is the same so we want to eliminate the y values. We must subtract one equation from the other as both y values have the same sign.
$(2) - (1)$
$$5x + y = 5$$
$$3x + y = 7$$
$$2x \quad = -2$$
Therefore, $x = -1$.
Substitute this into equation (1)
$$3 \times -1 + y = 7$$
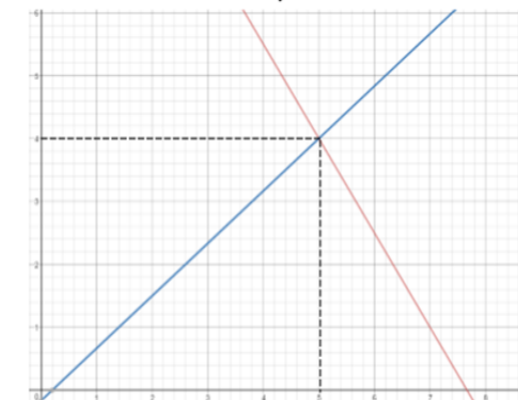$$-3 + y = 7$$
$$y = 10$$
Check your answers by substituting both values into the other equation.
$$5 \times -1 + 10 = 5$$

Solve the simultaneous equations:
$$3x + 2y = 23$$
$$5x - 6y = 1$$

The solution to these simultaneous equations is given by the point where the lines meet.
The x-coordinate gives the solution of x and the y-coordinate gives the solution of y.
In this example, the solution is $x = 5$ and $y = 4$.

Assessment style question

1. Solve the simultaneous equations

$$5x + 3y = 41$$
$$2x + 3y = 20$$

Do not use trial and improvement

Solve the simultaneous equations

$$2x + 4y = 26$$
$$3x - y = 4$$

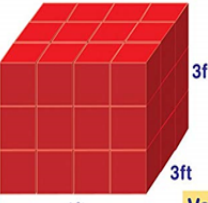Do not use trial and improvement

Volume

Key vocabulary

Volume
Cross section
Length
Width
Height
Pi (π)
Circumference
Diameter
Radius
Units^3

Picture perfect

Volume is the space inside a 3D shape.

Volume is the number of cubic units that fill up a solid figure.

$V = \ell \times w \times h$
$V = 4 \times 3 \times 3$
$V = 36$

Volume = 36 cubic feet
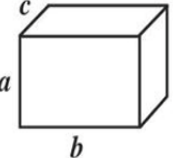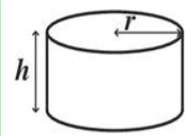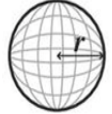
Volume formula

height (h)
width (w)
length (ℓ)
Volume = length x width x height
Volume = ℓ x w x h

Always remember

Volume units include mm^3, cm^3, m^3 etc.

CUBE	CUBOID	CYLINDER
$A = 6l^2$ $V = l^3$	$A = 2ab + 2ac + 2bc$ $V = abc$	$A = 2\pi r(r + h)$ $V = \pi r^2 h$

SPHERE	HEMISPHERE	TRIANGULAR PRISM
$A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$	$A = 2\pi r^2$ $V = \frac{2}{3}\pi r^3$	$V = \frac{1}{2}bhl$

CONE	FRUSTRUM	PYRAMID
$A = \pi r^2 + \pi rs$ $V = \frac{1}{3}\pi r^2 h$	$V = \frac{1}{3}\pi h(r^2 + rR + R^2)$	$V = \frac{1}{3}$ base area x h

Assessment style question

The cuboid and the triangular prism have the same volume. Find x.
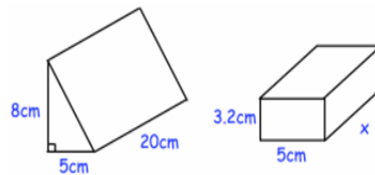
22 A frustum is made by removing a small cone from a large cone as shown in the diagram.

Volume of cone $= \frac{1}{3}\pi r^2 h$

The frustum is made from glass.
The glass has a density of $2.5\,g/cm^3$.
Work out the mass of the frustum.
Give your answer to an appropriate degree of accuracy.

A solid sphere fits perfectly inside a cube box of side length 10cm.
What percentage of the box is empty?

Quadratic equations

Key vocabulary

Quadratic equations
Binomial
Expand
Simplify
Factorise
Difference of two squares
Rearrange formulae

Factorising
$= 8n^2 + 20n$
$4n(2n + 5)$
Highest Common Factor

Always remember

Solving:
- Factorising
- Formula
- Completing the square
- Drawing a graph

Quadratic Equations
$ax^2 + bx + c$

Factorising:
easy...
$x^2 + 7x + 12 = 0$
$(x + 3)(x + 4) = 0$
$x = ^-3$ or $x = ^-4$

brackets
... more difficult!

$3x^2 - 5x + 2$
$3x^2 - 3x - 2x + 2$
$3x(x - 1) - 2(x - 1)$
$(3x - 2)(x - 1)$

multiply
6
1x6
2x3

Completing the square:
$x^2 + 4x - 3 = 0$
$(x + 2)^2 - 4 - 3 = 0$
$(x + 2)^2 - 7 = 0$
$x + 2 = \pm\sqrt{7}$
$x = \pm\sqrt{7} - 2$

half of 4x
subtract 2^2

Difference of Two Squares:
$x^2 - 16$
$(x - 4)(x + 4)$
x squared subtract 4 squared

Graphs:
Parabola - u shaped graph
draw lines to find solutions

The formula:
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Assessment style question

Make **p** the subject of the following equation:

$$m = 2p - 5$$

A) $p = \dfrac{m}{2} + 5$ C) $p = \dfrac{m+5}{2}$

B) $p = 2m - 5$ D) $p = \dfrac{m-5}{2}$

3. (i) Factorise $x^2 - 13x + 36$

(ii) Hence, or otherwise, solve the equation $x^2 - 13x + 36 = 0$

Scatter graphs

Key vocabulary

Scatter graph
Plot
Positive correlation
Negative correlation
No correlation
Line of best fit
Interpolate
Extrapolate

Picture perfect

Summary

A Scatter Graph helps us to determine whether there is a link between two sets of data.

If a graph has a 'direction' we can fit a Line of Best Fit to it.

If there is a link we call it a Correlation.

These three graphs show example of;

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Making Predictions — An extrapolation is a prediction beyond the given data points

Making Predictions — An interpolation is a prediction between given data points

Assessment style question

5. Make a scatter plot with a proper title (at the bottom) with the data and answer the question using hypothesis test.

The yearly data have been published showing the number of releases for each of the commercial movie studios and the gross receipts for those studios thus far. Based on these data, can it be concluded that there is relationship between the number of releases and the gross receipts?

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Gross receipts y (millions $)	3744	2062	1571	1664	214	301	258	134	155

Plot the following scatter graph and estimate a line of best fit.

Height, cm	157	160	148	160	177	156	166	170
Weight, kg	53	60	44	53	54	60	54	70

Quadratic Equations

Key vocabulary

Equation
Quadratic
Factorise
Formula
Substitute
Coefficient
Complete the square

Picture perfect

Positive quadratic 'u' shape ☺ x^2

Negative quadratic 'n' shape ☹ $-x^2$

Always remember

An equation where the highest power of the variable is 2

$$ax^2 + bx + c$$

Factorising $a = 1$ Quadratics

Aim: Convert quadratic into double brackets $(x \pm \quad)(x \pm \quad)$

Sum and product rule

$x^2 + bx + c$

Add to make b Multiply to make c
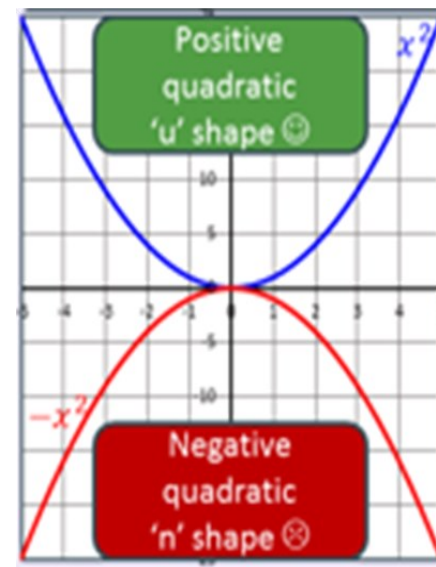
Example

$x^2 - 7x + 12$

Positive c → Signs Same
Negative b → Both Minus

Establish Signs

If c is positive — Signs are same
$x^2 + 5x + 6$ $(x + 3)(x + 2)$

If c is negative — Signs are different
$x^2 + 5x - 6$ $(x + 6)(x - 1)$

$(x - \quad)(x - \quad)$

Factors of 12 Which pair make 7?
12×1
6×2
4×3 $(x - 4)(x - 3)$

Factorising $a \neq 1$ Quadratics

$$(?x \pm \quad)(?x \pm \quad)$$

Factors of a to find possible values

$5x^2 - 14x - 3$
$= (5x \pm \quad)(1x \pm \quad)$

$6x^2 + x - 2 = \begin{array}{l}(3x \pm \quad)(2x \pm \quad) \\ (6x \pm \quad)(1x \pm \quad)\end{array}$ OR

Then find factors of c and see which satisfy b

Difference of Two Squares (DOTS)

$$a^2 - b^2 = (a + b)(a - b)$$
$$x^2 - 81 = (x + 9)(x - 9)$$
$$4y^2 - 25 = (2y + 5)(2y - 5)$$

The quadratic formula

The formula you **need** to know

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute values into the formula to generate two answers for x

$5x^2 + 8x - 4$ Identify values of a, b and c
$\quad a \quad\quad b \quad\quad c$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(5)(-4)}}{2(5)}$$

⬇ Substitute and simplify

$$x = \frac{-8 \pm \sqrt{144}}{10}$$

⬇ Carry out two calculations

$$x = 0.4 \text{ or } -2$$

Assessment style question

1. Find the roots of the quadratic equations by using the quadratic formula in each of the following:

   (i) $2x^2 - 3x - 5 = 0$ (ii) $5x^2 + 13x + 8 = 0$

   (iii) $-3x^2 + 5x + 12 = 0$ (iv) $-x^2 + 7x - 10 = 0$

   (v) $x^2 + 2\sqrt{2}x - 6 = 0$ (vi) $x^2 - 3\sqrt{5}x + 10 = 0$

   (vii) $\frac{1}{2}x^2 - \sqrt{11}x + 1 = 0$

2. Find the roots of the following quadratic equations by the factorisation method:

   (i) $2x^2 + \frac{5}{3}x - 2 = 0$ (ii) $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

   (iii) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$ (iv) $3x^2 + 5\sqrt{5}x - 10 = 0$
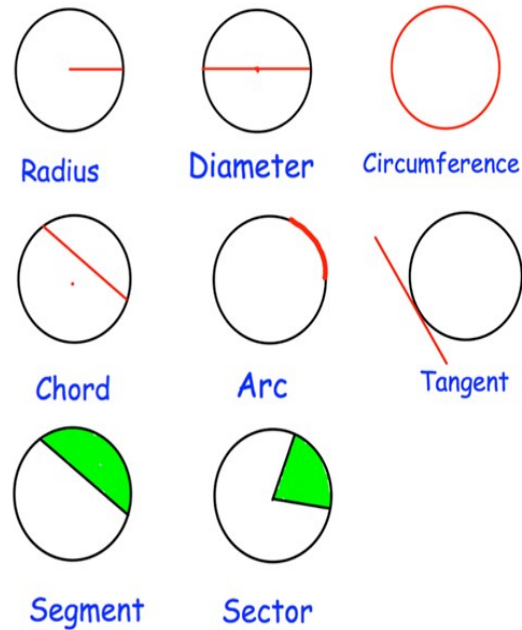
   (v) $21x^2 - 2x + \frac{1}{21} = 0$

Equation of a circle

Key vocabulary

Equation
Circle
Radius
Origin
Gradient
Intercept
Reciprocal
Tangent

Picture perfect

Parts of a Circle

Radius — Diameter — Circumference

Chord — Arc — Tangent

Segment — Sector

Always remember
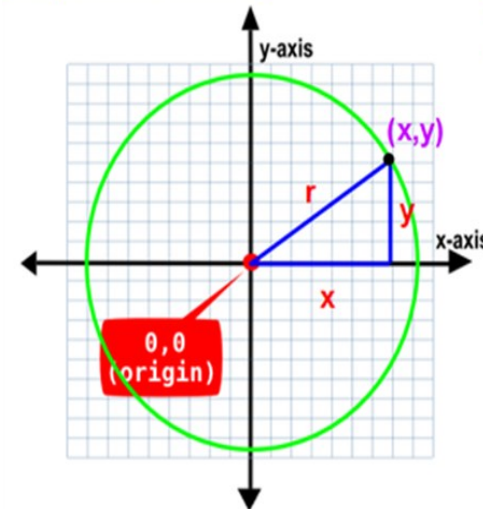
The equation of a circle centered at the origin

y-axis

(x,y)

r

y

x-axis

x

0,0 (origin)

$$x^2 + y^2 = r^2$$

Gradient of Tangent at a Point

Example 1: The circle with equation $x^2 + y^2 = 25$ passes through the point (3, 4). Find the equation of the tangent to the circle at that point.

(3, 4)

As this circle has an equation of the form $x^2 + y^2 = r^2$, it has centre (0, 0)

The gradient of the radius connecting (0, 0) and (3, 4) is — $\frac{4}{3}$

So the gradient, m, of the tangent at (3, 4) is — $\frac{-3}{4}$

The equation of the tangent has the form — $y - y_1 = m(x - x_1)$

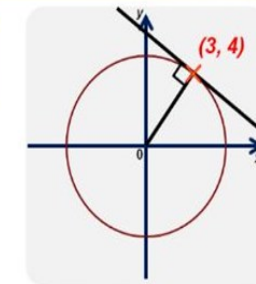At the point (3, 4), $x_1 = 3$ and $y_1 = 4$

So substituting these values gives — $y - 4 = \frac{-3}{4}(x - 3)$

Expand brackets — $y - 4 = \frac{-3}{4}x + \frac{9}{4}$

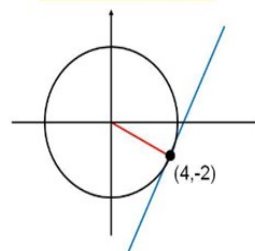Add 4 to both sides to give the equation of the tangent in terms of y — $y = \frac{-3}{4}x + \frac{25}{4}$

Key Stage 4 Maths Topics - Algebra

Assessment style question

How many mistakes can you find?

The graph shows a circle, centre (0,0)

gradient of radius $= \frac{4}{-2} = -2$

so gradient of tangent $= \frac{1}{2}$

substitute (4,-2) into $y = \frac{1}{2}x + c$

$\Rightarrow 4 = -1 + c$

$\Rightarrow c = 5$

$$y = \frac{1}{2}x + 5$$
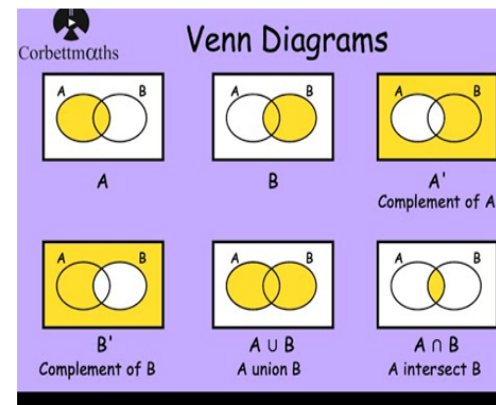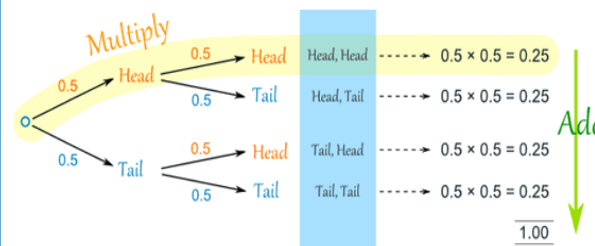
(4,-2)

Find the equation of a tangent to the circle at (4,-2)

Probability

Key vocabulary

Probability
Random
Likely
Impossible
Certain
Relative frequency
Probability scale
Unbiased
Theoretical probability
Sample space
Two way table
Tree diagram
Venn diagram
Dependant
Independent
Conditional probability

Picture perfect

Venn Diagrams

A
B
A'
Complement of A

B'
Complement of B
A ∪ B
A union B
A ∩ B
A intersect B

Assessment style question

19 Wendy goes to a fun fair.
She has one go at Hoopla.
She has one go on the Coconut shy.

The probability that she wins at Hoopla is 0.4
The probability that she wins on the Coconut shy is 0.3

(a) Complete the probability tree diagram.

(b) Work out the probability that Wendy wins at Hoopla and also wins on the Coconut shy.

Always remember

$P(\text{Event}) = \dfrac{number\ of\ successful\ outcomes}{total\ number\ of\ outcomes}$

$P(3) = \dfrac{2}{8} \Rightarrow \dfrac{1}{4}$

Simplify answers where possible

The 'OR' rule (mutually exclusive)

$P(a\ or\ b) = P(a) + P(b)$

$P(2\ or\ 4) = \dfrac{2}{8} + \dfrac{1}{8} \Rightarrow \dfrac{3}{8}$

Add each probability

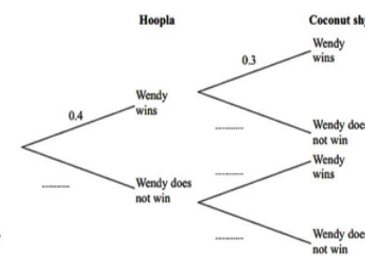The 'AND' rule (independent)

$P(a\ and\ b) = P(a) \times P(b)$

Flip a coin twice

$P(2\ tails) = \dfrac{1}{2} \times \dfrac{1}{2} \Rightarrow \dfrac{1}{4}$

Multiply each probability