Maths - Foundation

Algebra and graphs

Key vocabulary

Linear = A straight line graph

Quadratic = Means that the graph has an x2

Quadratic graph = curved graph

Parabola = A U shape Intercept = Where a graph crosses the y axis

Gradient = How steep a graph is

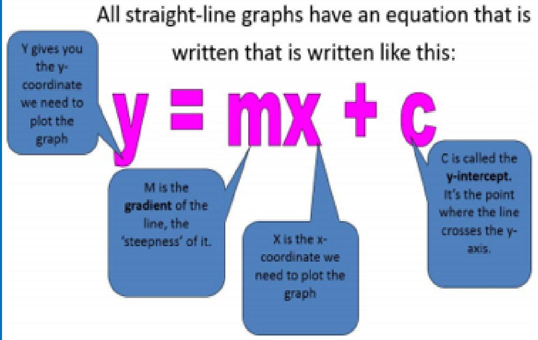
Midpoint = The centre or middle of a graph

Vertical = From top to bottom

Horizontal = From side to side

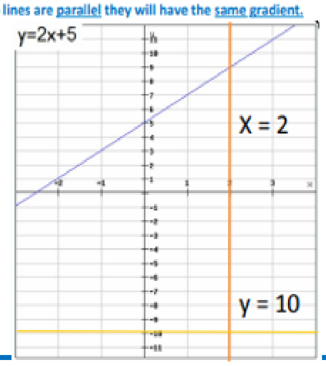
Picture perfect

All straight-line graphs have an equation that is written that is written like this:



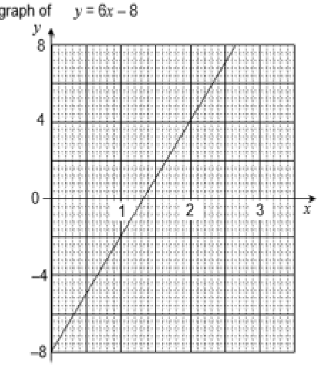
So the graph of y = 2x + 5 will cross the y axis at + 5 and will have a gradient of 2.

Key Point - If two lines are parallel they will have the same gradient.



Assessment style question

Here is part of the graph of $y = 6x - 8$



2 (a) Use the graph to work out the value of y when $x = 2$
Show on the graph how you obtained your answer.

2 (b) Use the graph to solve the equation $6x - 8 = 5$
Give your answer to 1 decimal place.
Show on the graph how you obtained your answer.

Always remember

Plotting graphs

You maybe asked to plot (draw) a graph using its equation,
e.g. plot the graph of y = 2x + 2
If x was 1 it would be 2 x 1 + 2 = 4
We use a table to help us do this.

x	0	1	2
2x	2 x 0 = 0	1 x 1 = 2	2 x 2 = 4
2x+2	0 + 2 = 2	2 + 2 = 4	4 + 2 = 6

Drawing graphs

On my graph now my coordinates are the circled numbers:
(0, 2) (1, 4) (2, 6)
I would now plot these coordinates and label my graph y = 2x + 2

Gradient

The gradient can be drawn or calculated by going one square across and seeing how many squares you go up or down. For example on the blue graph on the left, from 5 on the y axis, go across 1 and you will see it goes up 2 squares; so the gradient is 2.
The orange graph is x = 2 because the coordinates of any point on the line will be (2, ?) for example, (2, 1) (2, 2) (2, 3) (2, 4)
The yellow graph shows y = 10 because the coordinates of any point on the line will be (?, 10) e.g. (1, 10) (2, 10) (3, 10) (4, 10)

Coordinates

Coordinates are points on a graph. They are always written in brackets, separated by a comma, e.g. (1, 2). The first coordinate is the x, and the second is the y (x, y).
You always plot coordinates by going 'along the corridor and up the stairs'
So along the x axis to plot the first number and then up the y axis for the second number.

Draw the graph of the equation $y = x^2 + 3x$.

First, draw up a table of values for x, then substitute each value of x into x^2 and $3x$ to determine the y-value.

x	−4	−3	−2	−1	0	1	2
x^2	16	9	4	1	0	1	4
$3x$	−12	−9	−6	−3	0	3	6
$y = x^2 + 3x$	4	0	−2	−2	0	4	10

Maths - Foundation

Sketching Graphs

Key vocabulary

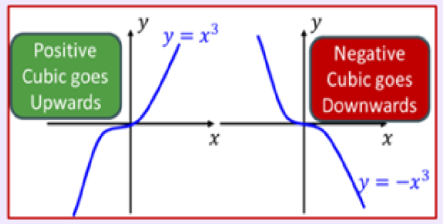
Reciprocal
Linear
Cubic
Quadratic
Substitute
Vertex
Symmetry
Intercept

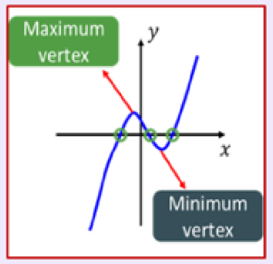
Picture perfect

Cubic graphs

Can be defined as an equation where the highest power of the variable (usually x) is 3

$$y = x^3 + 3x^2 + 5x - 20$$





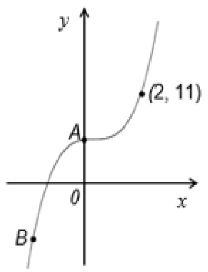
Assessment style question

This is a sketch of $y = x^3 + k$
The graph passes through (2, 11)

Work out the y-coordinate of point A.

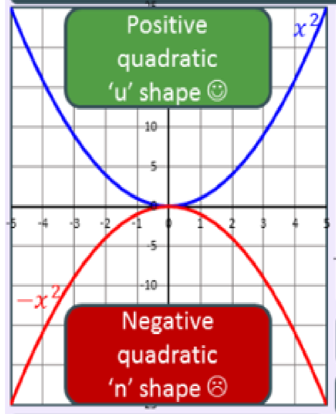
Point B has an x-coordinate of –2

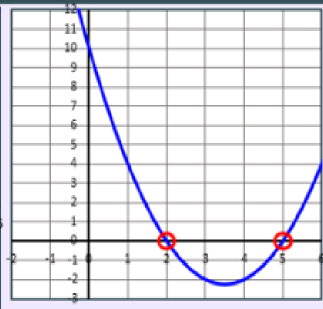
Work out the y-coordinate of point B.



Always remember

Quadratic graphs





The points where the graph crosses the x axis are known as roots.

These are the solutions to the quadratic when it equals zero

To plot graphs

Substitute the x values into equation to get y and plot points like coordinates

Maximum and minimum points

The line of symmetry runs through these points

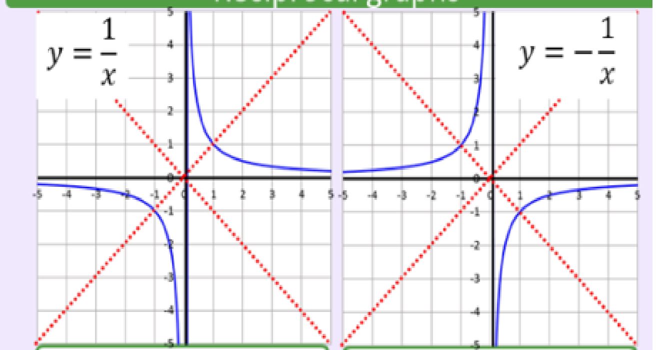
Read off coordinate point from graph

Find the midpoint of the roots and substitute into equation to calculate y – coordinate

Complete the square in the form $(x - p)^2 + q$

Max/min point can be found by ⇨ (p, q)

Reciprocal graphs



x-axis and y-axis are asymptotes

Two lines of symmetry

Maths - Foundation

Direct and Inverse Proportion

Key vocabulary

Direct
Inverse
Proportion
Equation
Constant
Proportional
Rate of change

Picture perfect

Direct Proportion

As one value increases, the other increases at the same rate

Three Coffees cost £7.50,
How much would five Coffees cost?

Find the value of one coffee then multiply by quantity needed

$£7.50 \div 3 = £2.50 \ per \ coffee$
$£2.50 \times 5 = \underline{£12.50}$

Inverse Proportion

As one value increases, the other decreases at the same rate

It takes 3 men 4 days to build a wall.
How long would it take 2 men?

Find the time taken by one man then divide by quantity stated

$3men \times 4 \ days = 12 \ days$
$12 \ days \div 2 \ men = \underline{6 \ days}$

Always remember

Direct Proportion

y is directly proportional to x

$y \propto x$ Constant of proportionality

$y = k \times x$ k is the rate of change

Solving direct proportion problems

p is directly proportional to t.
$p = 24, t = 8$
a) Find p when $t = 7$
b) Find t when $p = 39$

Compare two values

$p = k \times t$ ⇨ $24 = k \times 8$

Work out the value of k

$24 = k \times 8$ ⇨ ($\div 8$) $\frac{24}{8} = k$ ⇨ $3 = k$

Form equation to solve problems

$p = 3 \times t$ $a) \ p = 3 \times 7 = \underline{21}$

$b) \ 39 = 3 \times t$ ⇨ ($\div 3$) $t = \underline{13}$

Inverse Proportion

y is inversely proportional to x

$y \propto \frac{1}{x}$ $y = \frac{k}{x}$ Constant of proportionality k

Solving inverse proportion problems

p is inversely proportional to t.
$p = 16, t = 2$
a) Find p when $t = 8$
b) Find t when $p = 64$

Compare two values

Work out the value of k

$p = \frac{k}{t}$ $16 = \frac{k}{2}$ ⇨ ($\times 2$) $32 = k$

Form equation to solve problems

$p = \frac{32}{t}$ $a) \ p = \frac{32}{8} = \underline{4}$

$b) \ 64 = \frac{32}{t}$ ⇨ $t = \frac{32}{64} = \underline{0.5}$

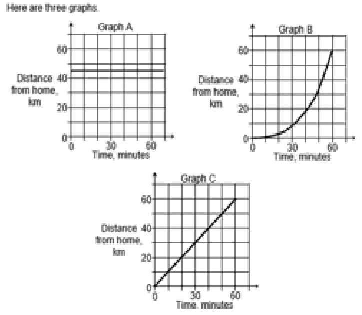
Assessment style question

$y = \frac{k}{x^2}$

When $x = 4$, $y = \frac{1}{2}$

Work out the value of y when $x = 2$

Here are three graphs.



Match each graph to a car.

Graph .... shows a car travelling at a steady spe

Graph .... shows a car that is not moving.

Graph .... shows a car whose speed is gradually increasing.

Maths - Foundation

Trigonometry

Key vocabulary

Hypotenuse: The longest side in a right angled triangle.
Opposite: The side facing the angle in a right angled triangle.
Adjacent: The side next to the angle given in a right angled triangle.
Square number: The result when you multiply a number by itself.
Inverse operation: The operation that reverses the effect of another operation.
Sine, Cosine, Tangent: Trigonometric ratios, relating to buttons on the calculator.

Picture perfect

SOH

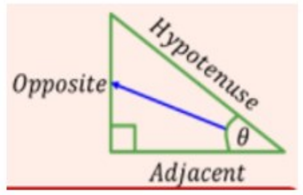
SINE = $\frac{OPP}{HYP}$

CAH

COSINE = $\frac{ADJ}{HYP}$

TOA

TANGENT = $\frac{OPP}{ADJ}$



Assessment style question

A helicopter leaves A and flies 40 miles due east. Then the helicopter flies 10 miles due south and arrives at B. Work out the bearing of B from A.

A boat leaves a port and sails 55km due west and then 30km due north and arrives at an oil rig. What is the bearing of the oil rig from the port?

Always remember

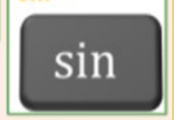
Trigonometry is used to calculate sides lengths and angles in triangles using three important ratios. Sine, Cosine and Tangent

Sine Function

Work out side lengths and angles when given the **opposite** and **hypotenuse**

$sin\theta = \frac{opposite}{hypotenuse}$

$\sin^{-1}$



$sin\theta$	Value
0°	0
30°	0.5
45°	$\frac{\sqrt{2}}{2}$= 0.707
60°	$\frac{\sqrt{3}}{2}$=0.866
90°	1

To get $\sin^{-1}$ you have to press shift first

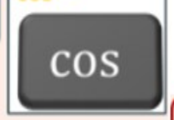
Only used to calculate an angle

Cosine Function

Work out side lengths and angles when given the **adjacent** and **hypotenuse**

$cos\theta = \frac{adjacent}{hypotenuse}$

$\cos^{-1}$



$cos\theta$	Value
0°	1
30°	$\frac{\sqrt{3}}{2}$= 0.866
45°	$\frac{\sqrt{2}}{2}$= 0.707
60°	0.5
90°	0

To get $\cos^{-1}$ you have to press shift first

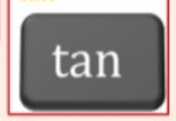
Only used to calculate an angle

Tangent Function

Work out side lengths and angles when given the **adjacent** and **opposite**

$tan\theta = \frac{opposite}{adjacent}$

$\tan^{-1}$



$tan\theta$	Value
0°	0
30°	$\frac{\sqrt{3}}{3}$
45°	1
60°	$\sqrt{3}$

To get $\tan^{-1}$ you have to press shift first

Only used to calculate an angle

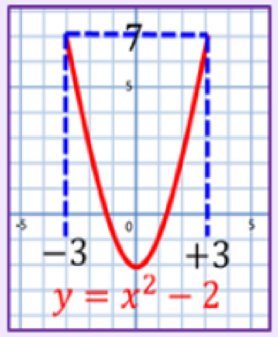
Solving Quadratic Equations

Key vocabulary

- **Solve**
- **Solution**
- **Plot**
- **Intercept**
- **Positive**
- **Quadratic**
- **Simultaneous**
- **Linear**

Picture perfect

Find the solutions of x when $x^2 - 2 = 7$



Plot graph and read off points

$x = 3$
or
$x = -3$

Always remember

An equation where the highest power of the variable is 2

$ax^2 + bx + c$

Factorising $a = 1$ Quadratics

Aim: Convert quadratic into double brackets $(x \pm \quad)(x \pm \quad)$

Sum and product rule

$x^2 + bx + c$

Add to make b — Multiply to make c

Establish Signs

If c is positive — Signs are same

$x^2 + 5x + 6 \quad (x + 3)(x + 2)$

If c is negative — Signs are different

$x^2 + 5x - 6 \quad (x + 6)(x - 1)$

Example

$x^2 - 7x + 12 \rightarrow (x - \quad)(x - \quad)$

Positive c → Signs Same

Negative b → Both Minus

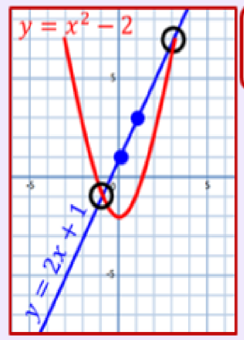
Factors of 12: 12×1, 6×2, 4×3

Which pair make 7? $(x - 4)(x - 3)$

$y = x^2 - 2$

$y = 2x + 1$

Solve simultaneously or **graph** each equation



Two intersections = two sets of solutions

$x = 3 \quad y = 7$

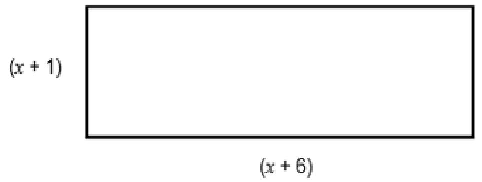
or

$x = -1 \quad y = -1$

Assessment style question

Solve $x^2 - 11x + 30 = 0$

6 The area of the rectangle is 66 cm²



6 (a) Using this information, show that $x^2 + 7x - 60 = 0$

Maths - Higher

Sketching Graphs

Key vocabulary

Reciprocal
Linear
Cubic
Quadratic
Substitute
Vertex
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Picture perfect

Cubic graphs

Can be defined as an equation where the highest power of the variable (usually x) is 3

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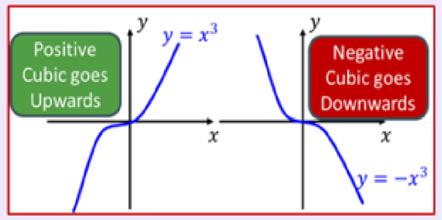

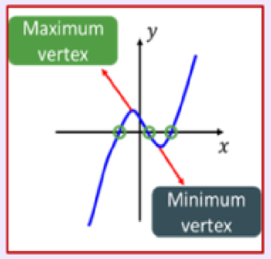

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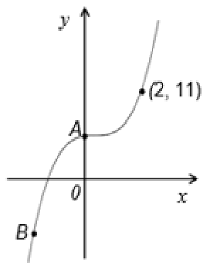


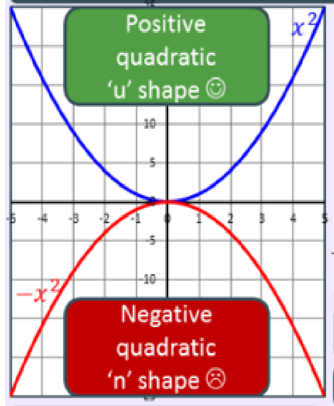
Work out the y-coordinate of point A.

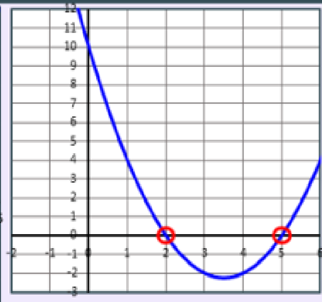
Point B has an x-coordinate of –2

Work out the y-coordinate of point B.

Always remember

Quadratic graphs





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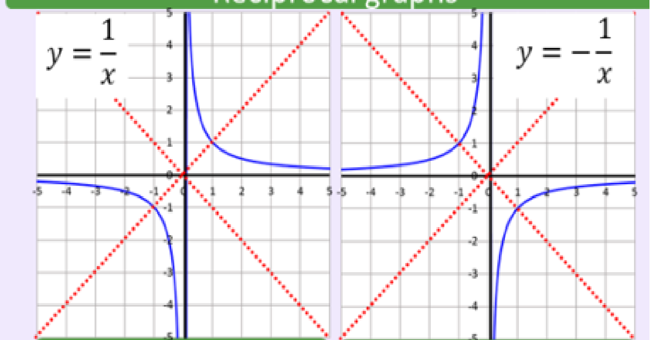
Reciprocal graphs



x-axis and y-axis are asymptotes

Two lines of symmetry

Maths - Higher

Direct and Inverse Proportion

Key vocabulary

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Inverse
Proportion
Equation
Constant
Proportional
Rate of change

Picture perfect

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Compare two values

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Work out the value of k

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Form equation to solve problems

$p = 3 \times t$ — a) $p = 3 \times 7 = \underline{21}$

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$y \propto \frac{1}{x}$ → $y = \frac{k}{x}$ — Constant of proportionality k

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Form equation to solve problems

$p = \frac{32}{t}$ — a) $p = \frac{32}{8} = \underline{4}$

b) $64 = \frac{32}{t}$ ⇨ $t = \frac{32}{64} = \underline{0.5}$

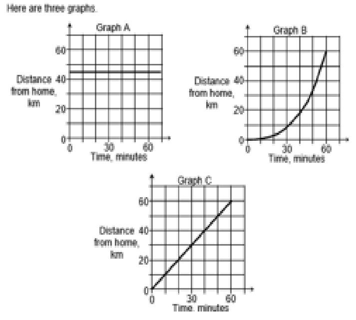
Assessment style question

$y = \frac{k}{x^2}$

When $x = 4$, $y = \frac{1}{2}$

Work out the value of y when $x = 2$

Here are three graphs.



Match each graph to a car.

Graph .... shows a car travelling at a steady spe

Graph .... shows a car that is not moving.

Graph .... shows a car whose speed is gradually increasing.

Maths - Higher

Inequalities

Key vocabulary

Inequality: The relationship between two expressions that are not equal

Integer: A whole number

Solve: Find a numerical value that satisfies the equation or inequality

Inverse operation: The operation that reverses the effect of another operation e.g. subtraction in the inverse of addition

Picture perfect

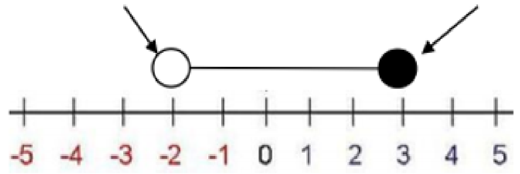
$x > y$	x is greater than y	$x < y$	x is less than y
$x \geq y$	x is greater than or equal to y	$x \leq y$	x is less than or equal to y

State the values of n that satisfy:

$-2 < n \leq 3$

Cannot be equal to 2   Can be equal to 3

Cannot be equal to 2   Can be equal to 3



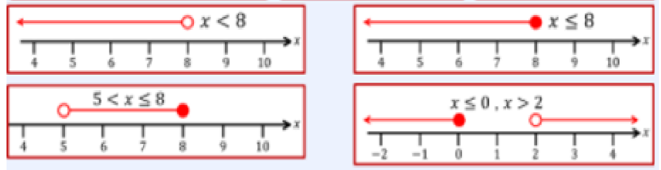
Always remember

Inequalities on a numberline

Shows the range of values for an inequality

Open circle does **not include** value

Closed circle **includes** value



Solving Linear inequalities

Same process as solving linear equations

When we multiply or divide by a **negative number**, we must **flip** the inequality sign

$x + 5 \leq 16$

$x \leq 16 - 5$

$x \leq 11$

$-2x > 4$

$x < -2$

$4 < 3x + 1 < 16$

$1 < x < 5$

Solving Quadratic inequalities

Same process as solving equations

Sketch the graph to interpret the range of values required.

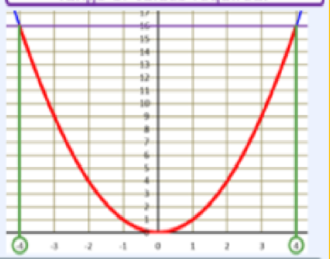
$x^2 \leq 16$

$x = \pm 4$

The roots

$-4 \leq x \leq 4$

The range of values below the line



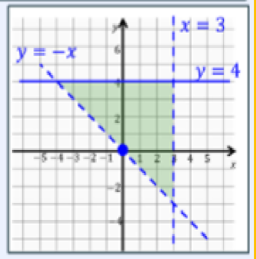
Two variable inequalities

Inequalities with 2-variables need to be represented on a graph

Shade the region satisfied by the inequalities: $y > -x, y \leq 4, x < 3$

Draw the line graph for each inequality

< - Dashed line   ≤ - Solid line



Assessment style question

Question 4: On copies of the grid below, clearly indicate the region that satisfies the following inequalities.

(a) $y > x - 1$, $x \geq -2$ and $y < 2$

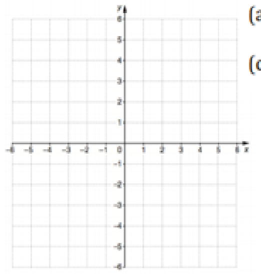
(b) $y \leq 2x$, $x \leq 2$ and $y > -4$

(c) $y \leq -2x + 2$, $x \geq 0$ and $y > x - 4$

(d) $x + y < 3$, $-2 \leq x < 3$ and $y \geq 0$

(e) $y \leq 5x - 4$, $y > x - 4$ and $y \leq -\frac{1}{2}x + 2$

(f) $y \leq -2x + 4$, $y < 2x - 6$ and $-4 < y < -3$



Question 5: Solve each of the inequalities below

(a) $4x + 3 > 2x + 11$ (b) $x + 1 \geq 3x - 18$

(c) $13x - 12 < 3x + 13$ (d) $7x - 5 \geq 3x + 11$

Maths - Higher

Pythagoras and Trigonometry

Key vocabulary

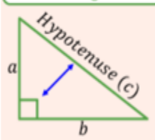
Hypotenuse: The longest side in a right angled triangle.
Opposite: The side facing the angle in a right angled triangle.
Adjacent: The side next to the angle given in a right angled triangle.
Square number: The result when you multiply a number by itself.
Inverse operation: The operation that reverses the effect of another operation.
Sine, Cosine, Tangent: Trigonometric ratios, relating to buttons on the calculator.

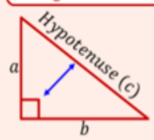
Picture perfect

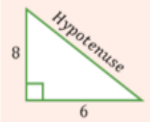
$$c^2 = a^2 + b^2$$

SOH SINE = $\frac{OPP}{HYP}$ **CAH** COSINE = $\frac{ADJ}{HYP}$ **TOA** TANGENT = $\frac{OPP}{ADJ}$

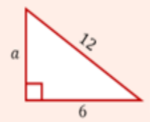
Finding the Hypotenuse	Finding the Shorter side
If you know the lengths of the **two shorter sides**, you can calculate the length of the hypotenuse.	If you know the **Hypotenuse** and a **shorter side**, you can calculate the length of the other shorter side.
Square the two sides → **Add** them → Square root for answer	Square the two sides → **Subtract** them → Square root for answer







$c^2 = 8^2 + 6^2$
$c^2 = 64 + 36$
$c^2 = 100$ ($\sqrt{\ }$)
$C = 10$



$c^2 - b^2 = a^2$
$12^2 - 6^2 = a^2$
$144 - 36 = a^2$
$108 = a^2$ ($\sqrt{\ }$)
$10.39 = a$

Always remember

Trigonometry is used to calculate sides lengths and angles in triangles using three important ratios. Sine, Cosine and Tangent

Sine Function

Work out side lengths and angles when given the **opposite** and **hypotenuse**

$sin\theta = \frac{opposite}{hypotenuse}$

sin^{-1}

$sin\theta$	Value
0°	0
30°	0.5
45°	$\frac{\sqrt{2}}{2}$= 0.707
60°	$\frac{\sqrt{3}}{2}$=0.866
90°	1

To get $\sin^{-1}$ you have to press shift first

Only used to calculate an angle

Cosine Function

Work out side lengths and angles when given the **adjacent** and **hypotenuse**

$cos\theta = \frac{adjacent}{hypotenuse}$

cos^{-1}

$cos\theta$	Value
0°	1
30°	$\frac{\sqrt{3}}{2}$=0.866
45°	$\frac{\sqrt{2}}{2}$=0.707
60°	0.5
90°	0

To get $\cos^{-1}$ you have to press shift first

Only used to calculate an angle

Tangent Function

Work out side lengths and angles when given the **adjacent** and **opposite**

$tan\theta = \frac{opposite}{adjacent}$

tan^{-1}

$tan\theta$	Value
0°	0
30°	$\frac{\sqrt{3}}{3}$
45°	1
60°	$\sqrt{3}$

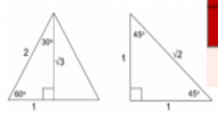
To get $\tan^{-1}$ you have to press shift first

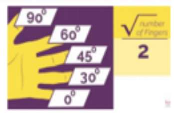
Only used to calculate an angle

Trigonometry – Exact values

For your exam you will need to learn the following values.
(Use the hand trick or triangles to help you learn them)

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	–





Assessment style question

A helicopter leaves A and flies 40 miles due east. Then the helicopter flies 10 miles due south and arrives at B. Work out the bearing of B from A.

A boat leaves a port and sails 55km due west and then 30km due north and arrives at an oil rig. What is the bearing of the oil rig from the port?

Growth and Decay

Key vocabulary

Percentage change
Simple interest
Compound interest
Growth/Decay

Picture perfect

Simple Interest

Follows the I=PRY formula

Interest ⇨ Principal × Rate × Years

Calculate the Simple Interest earned on £350 at a rate of 9% p.a for 4 years?

Interest ⇨ $£350 \times 0.09 \times 4 = £126$

Remember to convert the % to a decimal

To calculate other parts of the formula, you will need to change the subject

How many years would it take for £45000 to receive £19800 Simple Interest at a rate of 5.5% p.a?

$\frac{\text{Interest}}{\text{Principal} \times \text{Rate}}$ ⇨ Years

Always remember

Compound Growth

An amount is increased or decreased by a percentage

The process is repeated several times at each interval

The most efficient way to do this is using a Multiplier

The general formula for compound growth and decay

Interest ⇨ Principal × $(1 \pm \text{Rate})^{\text{Years}}$ (Growth: +, Decay: −)

£4000 is invested at a rate of 5% p.a for three years. Calculate Final value of the investment after three years.

$£4000 \times 1.05^3 = £4630.50$

A car worth £15000 depreciates in value at a rate of 15% p.a. What is the depreciated value of the car after 4 years

$£15000 \times 0.85^4 = £7830.09$

To calculate other parts of the formula, you will need to change the subject

Assessment style question

A tree is 80cm when planted.
Each year the height of the tree increases by 22%.
After how many complete years will the height of tree be at least 3m?

The number of polar bears in a region is decreasing by 5% per year.
There are 3000 polar bears in the region in 2017.
What year will be the first year with less than 1000 polar bears in the region?

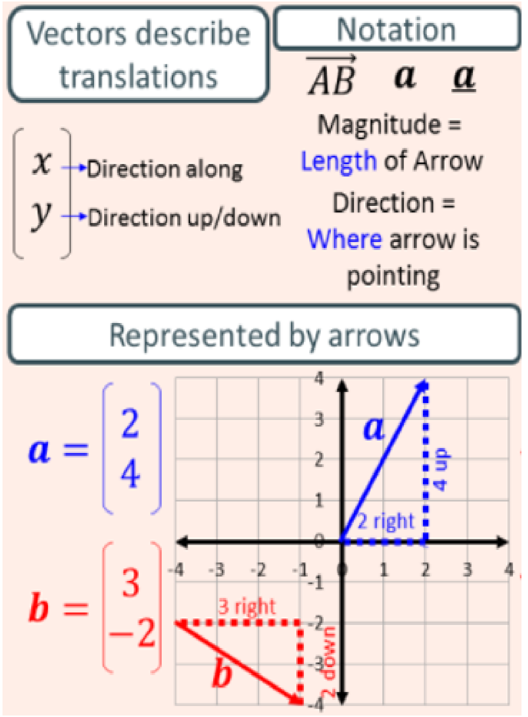
Maths - Higher

Vectors

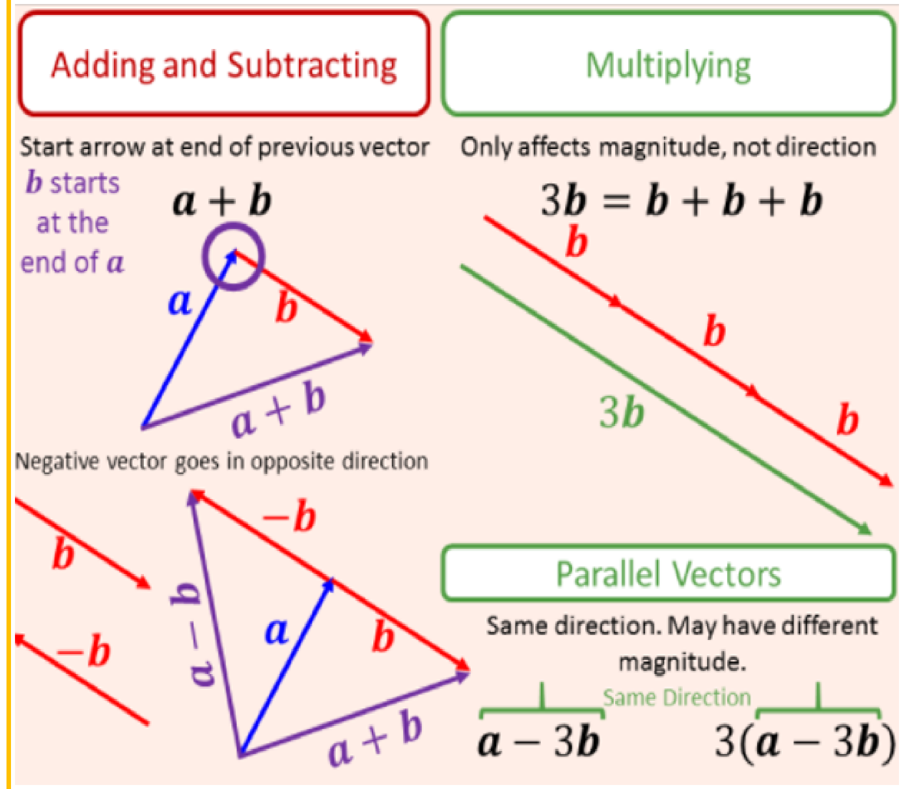
Key vocabulary

Scalar - A scalar is a non-vector quantity.
The word scalar is also used for a constant number in front of the vector.
Magnitude – Length of the vector
Parallel – Lines that are equidistant apart
Equidistant – Equal distance

Picture perfect



Always remember



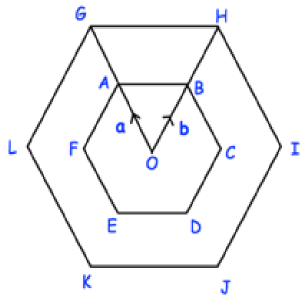
Assessment style question

The translation vector to take shape C to shape D is $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$

What translation vector takes shape D to shape C?

(a) Write the vector AB in terms of **a** and **b**.

(b) Write the vector $\overrightarrow{OG}$ in terms of **a** and **b**.



ABCDEF and GHIJKL are regular hexagons with centre O.
GHIJKL is an enlargement of ABCDEF, with scale factor 2.
$\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$

Maths - Higher

Transforming Functions

Key vocabulary

Translate
Compress
Stretch
Reflect
Symmetry
X-axis
Y-axis
Function

Picture perfect

y = f(x) + C	• C > 0 moves it up • C < 0 moves it down
y = f(x + C)	• C > 0 moves it left • C < 0 moves it right
y = Cf(x)	• C > 1 stretches it in the y-direction • 0 < C < 1 compresses it
y = f(Cx)	• C > 1 compresses it in the x-direction • 0 < C < 1 stretches it
y = −f(x)	• Reflects it about x-axis
y = f(−x)	• Reflects it about y-axis

Always remember

Translation

A translation can be defined as the movement 'sliding' of a shape to a new position

$f(x) + a$

The graph shifts up/down the $y - axis$ by a units

$f(x + a)$

The graph shifts left/right along the $x - axis$ by a units

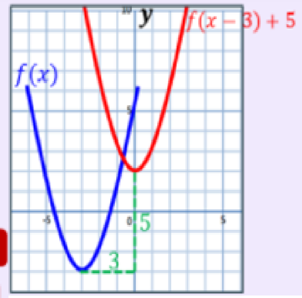
$+ =$ Left

$- =$ Right

Adding to the function causes a **Translation**

Given $f(x)$,
Sketch $f(x - 3) + 5$

3 units right   5 units up



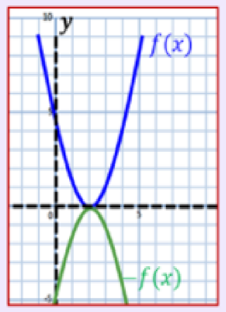
Reflection

Reflection: The replacement of each point on one side of a line by the point symmetrically placed on the other side of the line.

$y = -f(x)$

The **outputs** are reversed

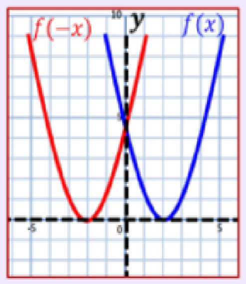
The graph is reflected in the $x - axis$



$y = f(-x)$

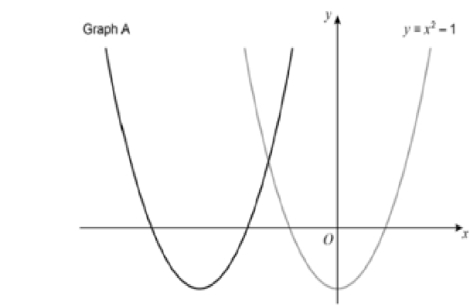
The **inputs** are reversed

The graph is reflected in the $y - axis$



Assessment style question

Here are sketches of two graphs.



The graph of $y = x^2 - 1$ is translated 3 units to the left to give graph A.

(a) The equation of graph A can be written in the form $y = x^2 + bx + c$

Work out the values of b and c.

Maths - Higher

Sine and Cosine Rule

Key vocabulary

Sine and Cosine rules: Rules used to find sides and angles in a triangle which is not right angled.

Formula

Substitution

Inverse function

Angle

Side

Picture perfect

The Sine rule formula

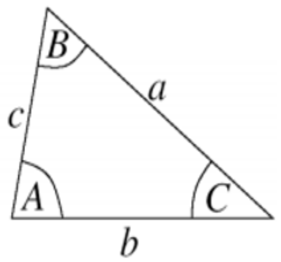
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Missing length — Missing angle

The Cosine rule formula

$a^2 = b^2 + c^2 - 2bc\cos A$ ⟹ Missing length

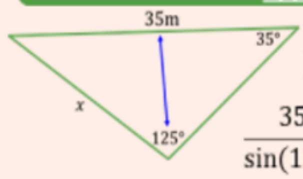
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ ⟹ Missing angle



Always remember

The Sine rule formula

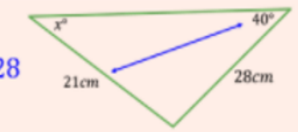
We tend to use the Sine rule if we know an **angle** and its **opposite length**



$\frac{35}{\sin(125)} = \frac{x}{\sin(35)}$ $\times \sin(35)$

$\frac{35}{\sin(125)} \times \sin(35) = x$ $\underline{24.51m}$

$\frac{\sin(x^\circ)}{28} = \frac{\sin(40)}{21}$ $\times 28$



$\underline{x = 59^\circ}$

$\sin(x) = \frac{\sin(40) \times 28}{21}$ $x^\circ = \sin^{-1}\left(\frac{\sin(40) \times 28}{21}\right)$

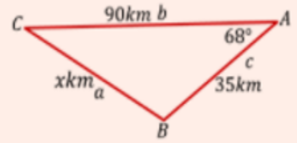
The Cosine rule formula

If we know two sides **AND** the included angle

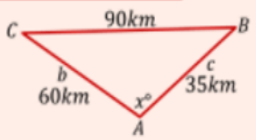
$x^2 = (90)^2 + (35)^2 - (2(90)(35)\cos(68^\circ))$

$x^2 = 6964.978\ldots$

$\underline{x = 83.46km}$



$\cos x = \frac{(60)^2 + (35)^2 - (90)^2}{2(60)(90)}$



$\cos x = -0.303\ldots$

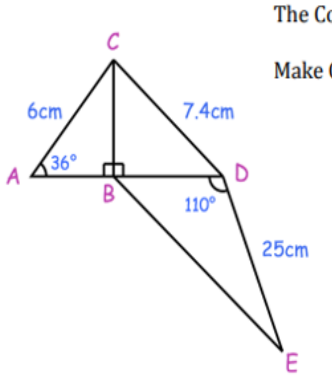
$x = \cos^{-1}(-0.303\ldots)$ $\underline{x = 107.7^\circ}$

Assessment style question

In the diagram:

ABD is a straight line.
AC = 6cm CD = 7.4cm DE = 25cm
Angle BAC = 36° Angle BDE = 110°

Calculate the length of BE



The Cosine Rule is $a^2 = b^2 + c^2 - 2bc\text{Cos}A$

Make CosA the subject.

Maths - Higher

Circle Theorems

Key vocabulary

Cyclic quadrilateral A quadrilateral with all four vertices on the circumference of a circle

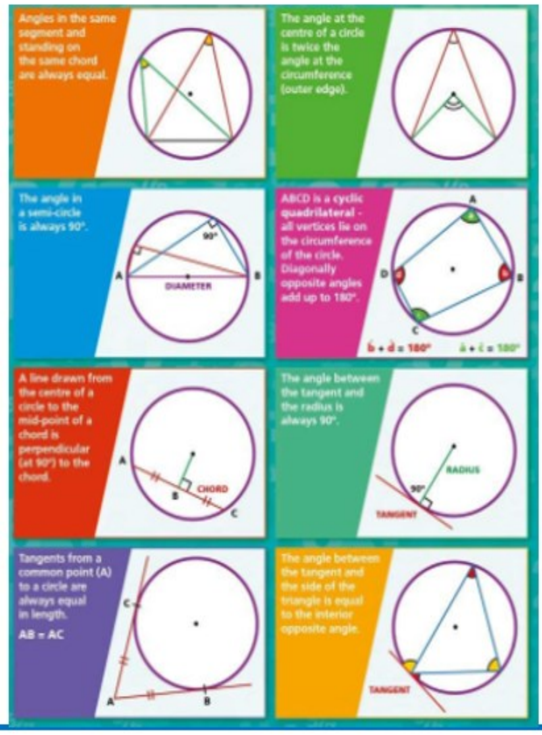
A **chord** is a line that cuts across a circle

The **perpendicular** from the centre of a circle to a chord **bisects the chord.**

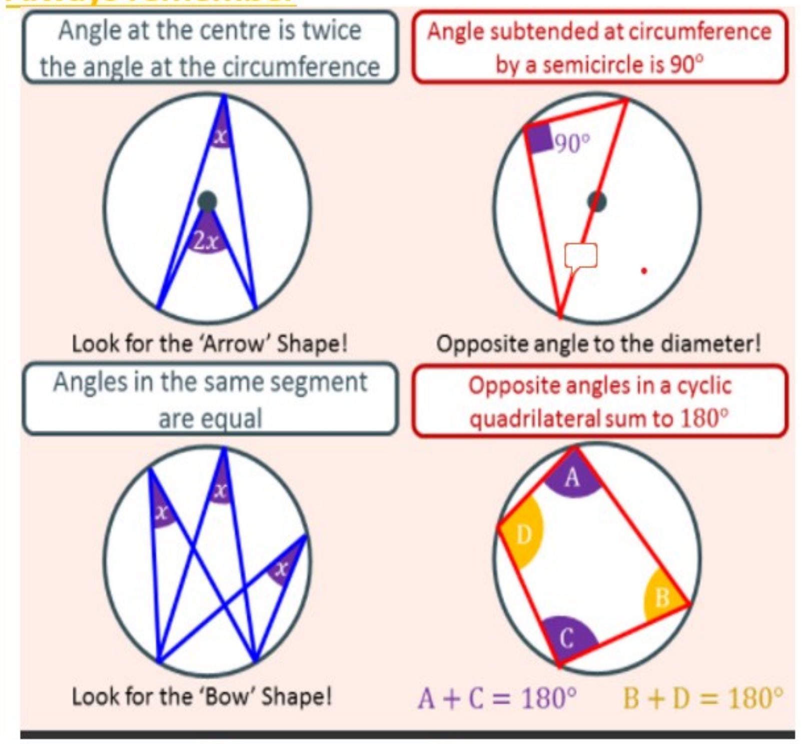
The line drawn from the centre of a circle to the midpoint of a chord is at **right angles to the chord**

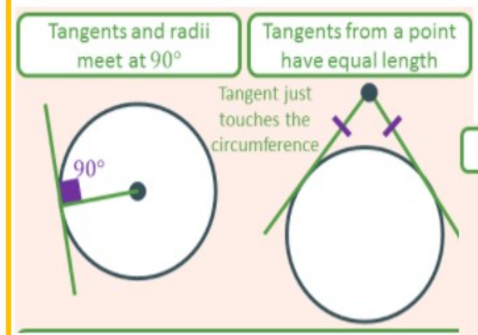
The triangle formed by two radii and a chord is **isosceles**

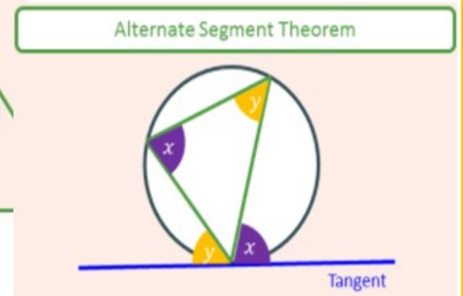
Picture perfect



Always remember



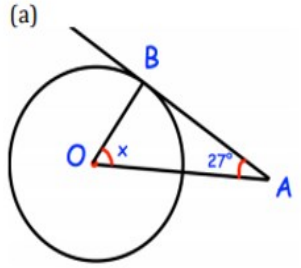




Assessment style question

Question 10: Find the value of x in each diagram. The lines AB and AC are tangents.

(a)



(b)