Maths - Foundation

Angles and Scale

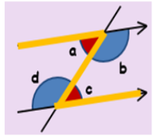
Key vocabulary

Angle, degree, calculate, parallel, polygon, perpendicular, isosceles, corresponding, alternate, co-interior, vertically opposite, bearing, scale, three figure bearing, north line

Deeper Learning

Next steps = Angles in polygons, trigonometry, similar shapes.

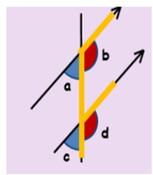
Picture perfect



Alternate angles are equal

Angle a = c and b = d

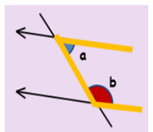
They make a Z shape



Corresponding angles are equal

Angle a = c and b = d

They make an F shape

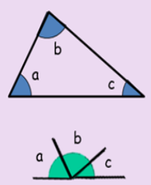


Supplementary angles add up to 180°

Angle a + b = 180°

They make a C shape

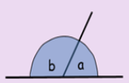
Angles in a triangle



Angles in a triangle add up to 180°

Angles a + b + c = 180°

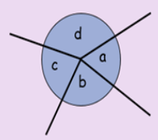
Angles on a straight line



Angles on a straight line add up to 180°

Angles a + b = 180°

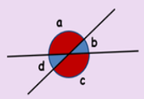
Angles around a point



Angles at a point add to 360°

Angle a + b + c + d = 360°

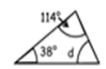
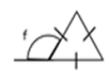
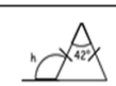
Vertically opposite angles



Vertically opposite angles are equal

Angle a = c and b = d

Always remember

Question	Answer	Reason
	150°	Angles on a straight line add up to 180°
	54°	Vertically opposite angles are equal
	150°	Angles around a point add up to 360°
	28°	Angles in a triangle add up to 180°
	40°	Angles in a triangle add up to 180°
	120°	Equilateral triangles have all equal angles and angles in a triangle add up to 180° and angles on a straight line add up to 180°
	58°	Angles on a straight line add up to 180° and angles in a triangle add up to 180°
	111°	Isosceles triangles have two equal angles and angles in a triangle add up to 180° and angles on a straight line add up to 180°

Maths - Foundation

Scale drawings

Key vocabulary

Angle, degree, calculate, parallel, polygon, perpendicular, isosceles, corresponding, alternate, co-interior, vertically opposite, bearing, scale, three figure bearing, north line

Picture perfect

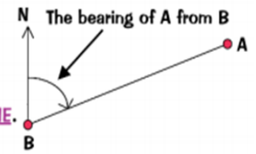
Bearings

To find or plot a bearing you must remember the three key words:

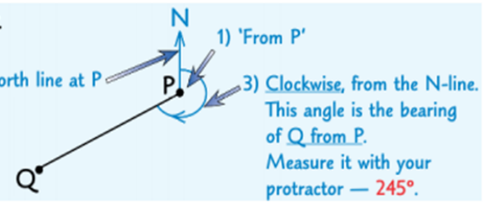
1) 'FROM' — Find the word 'FROM' in the question, and put your pencil on the diagram at the point you are going 'from'.

2) NORTH LINE — At the point you are going FROM, draw in a NORTH LINE. (There'll often be one drawn for you in exam questions.)

3) CLOCKWISE — Now draw in the angle CLOCKWISE from the north line to the line joining the two points. This angle is the required bearing.



Find the bearing of Q from P.

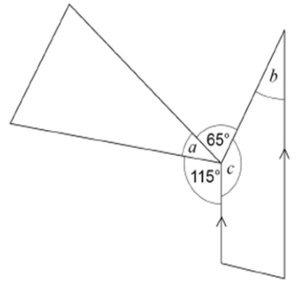


ALL BEARINGS SHOULD BE GIVEN AS 3 FIGURES
e.g. 176°, 034° (not 34°), 005° (not 5°), 018° etc.

° Angles

Assessment style question:

The diagram shows a triangle and a trapezium.



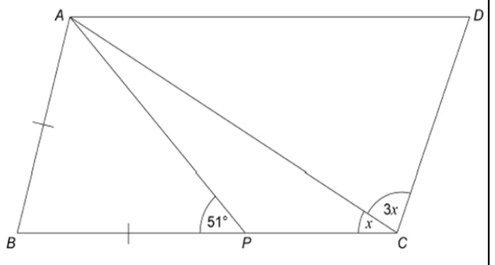
Not drawn accurately

Prove that $a = b$

$ABCD$ is a parallelogram.

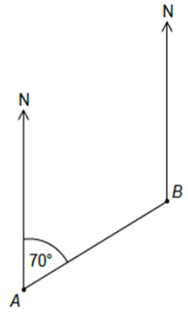
$AB = BP$

Not drawn accurately



Work out the size of angle x.

The bearing of B from A is 070°



Circle the bearing of A from B.

Maths - Foundation

Basic Number

Key vocabulary

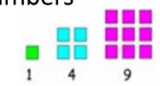
Integer, positive, negative, greater than, less than, inequality, place value, inverse, simplify, approximate, estimate, ascending, descending, powers of 10, factor, multiple, highest common factor, lowest common multiple, prime, square, square root, cube, cube root, prime factor decomposition.

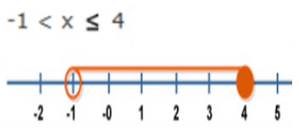
Deeper Learning

Next steps = Graphing inequalities and shading regions

Picture perfect

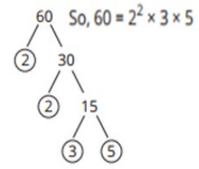
Square numbers





Writing a Number as a Product of its Prime Factors

Use a prime factor tree, but don't forget to write your final answer as a product (multiplied) using indices where necessary.



Types of Number

odd	end in 1, 3, 5, 7, 9
even	end in 0, 2, 4, 6, 8
prime	has exactly two factors, 1 and itself. E.g. 2, 3, 5, 7, 11...
factor	a number that divides exactly into another number, e.g. 3 is a factor of 9
highest common factor	the largest factor common to two or more numbers.
multiple	a number in the times table of another, e.g. 10 is a multiple of 5
lowest common multiple	the smallest number in two different times tables.

Always remember

Standard Form

This is always in the form $x \times 10^n$, where $1 \leq x < 10$.

For very big numbers, n will be positive and for very small numbers n will be negative.

Squares, Cubes and Roots

Square Numbers

Square numbers are found by multiplying a number by itself.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100...

The square root is the inverse, for example, the square root of 25 is 5.

Cube Numbers

Cube numbers are found by multiplying a number by itself, and multiplying by itself again.

1, 8, 27, 64, 125...

The cube root is the inverse, for example, the cube root of 64 is 4.

Mathematical Symbols

≠ is not equal to

< is less than

≤ is less than or equal to

> Is greater than

≥ is greater than or equal to

Estimation

To estimate a calculation, round all numbers to one significant figure.

E.g. Estimate 3.1 × 495

$3.1 \times 495 \approx 3 \times 500$

$3.1 \times 495 \approx 1500$

Place Value

Place value is the value of each digit in a number.

E.g. In the number 321, the 3 represents 300, the 2 represents 20 and 1 is the unit value.

Assessment style question: Diaries are sold in boxes of 12, Pencils are sold in boxes of 10, Rulers are sold in boxes of 6

A teacher wants to buy the same number of diaries, pencils and rulers. Work out the **smallest** number of boxes of each item he could buy.

Maths - Foundation

Factors and multiples

Key vocabulary

Multiples – The times tables of a specific number.

LCM – Lowest common multiple.

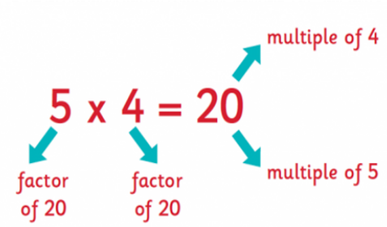
Factor – An integer which divides into another fully with no remainder.

HCF – Highest common factor.

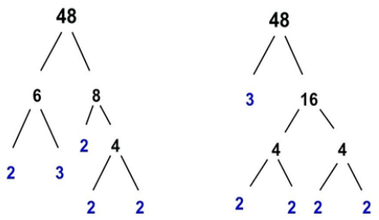
Prime number – An integer with only 2 factors.

Prime factorisation – Writing a number as a product of it's prime factors.

Picture perfect



Prime factorisation



48 = 2 x 2 x 2 x 2 x 3

Assessment style question

Mary is organising a charity hot dog sale.
There are 18 bread rolls in each packet.
There are 15 hot dogs in each packet.
Mary buys exactly the same number of bread rolls as hot dogs.

What is the smallest number of each packet that Mary can buy?

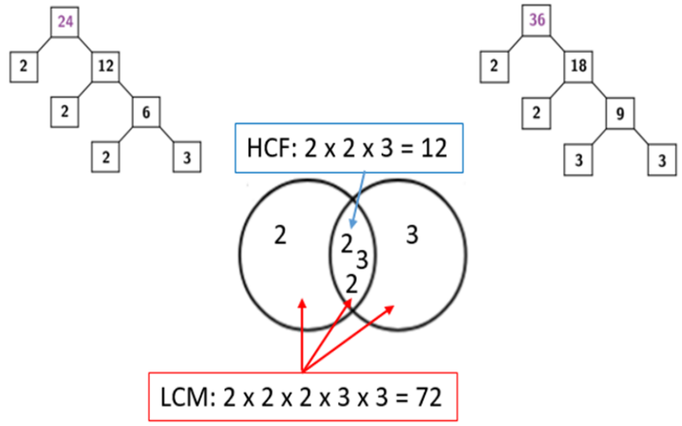
The Highest Common Factor (HCF) of two numbers is 6.
The Lowest Common Multiple (LCM) of the same numbers is 60.

What are the two numbers?

Always remember

HCF and LCM

Find the HCF and LCM of 24 and 36



2 is the only even prime number.

1 is NOT a prime number because it only has 1 factor.

Maths - Foundation

Basic Algebra

Key vocabulary

Powers, Indices, Reciprocal, Roots, Brackets, Multiply out, Expand, Simplify, Factor, Factorise, Algebra, Notation, Coefficient, Expression, Equation, Formula, Identity, Inequality, Term, Collecting like terms,

Deeper Learning

Next steps = Expanding two brackets, factorising quadratics, solving equations,

Picture perfect

Expanding brackets:

- Expanding means remove brackets
- Use the claw

e.g. $3(5a-2) = (3\times 5a) - (3\times 2)$

$= 15a - 6$

Factorising expressions:

- The opposite (inverse) of expanding
- Answer will include brackets
- Look for common factors (numbers and algebra)
- Always choose the HCF

e.g. 1 $10a + 15 = 5(2a+3)$

10 & 15 both in the 5 times table

$10a = 5\times 2a$ $\quad 15 = 5\times 3$

Always remember

Algebraic Notation

Algebra uses a shorthand system where variables (symbols) are represented by letters.

E.g. $a + a = 2a$

$2 \times b = 2b$

$c \times c = c^2$

$3d \times 2e = 6de$

Factorising Brackets

To factorise into one bracket, find the highest common factor for each term.

E.g. $4x + 10 = 2(2x + 5)$

Expanding Brackets

To expand one bracket:
make sure the term on the outside multiplies everything on the inside.

E.g. $4(2x - 3) = 8x - 12$

Simplifying expressions (adding/subtracting)

- Can only simplify like terms.
- Be sure to include the sign before the term

e.g. $2a + 3b - a + 4b = a + 7b$

$2a - a = a \quad +3b + 4b = +7b$

Term = 3a
Expression = 5x + 2
Equation = 5x – 3 = 47
Formula = C = πd
Identity= $10x + 4 \equiv 2(5x + 2)$

Assessment style question: Circle the correct words to complete each sentence:

(a)	$x^2 + 3x = 16$ is	an expression	an equation	a formula	an identity
(b)	$x^2 + 3x = 16$ is	an expression	an equation	a formula	an identity
(c)	$(x + 3)^2 \equiv x^2 + 6x + 9$ is	an expression	an equation	a formula	an identity

$(3x + 1)(x - 2) + ax + b \equiv 3x^2 + 8x - 5$

Work out the values of a and b.

Maths - Foundation

Fractions

Key vocabulary

Fraction – A quantity which is not a whole number.

Decimal – A decimal number is often used to mean a number that uses a decimal point followed by digits that show a value smaller than one.

Percentage – Amount out of one hundred.

Improper fraction - The numerator is larger than the denominator.

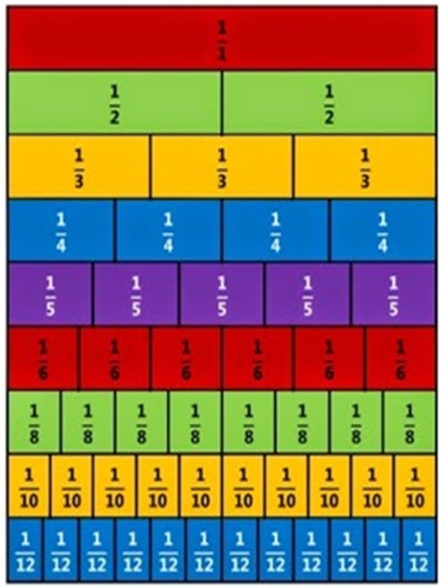
Mixed Number – A whole number and a fraction together.

Equivalent – When 2 amounts are equal they are equivalent

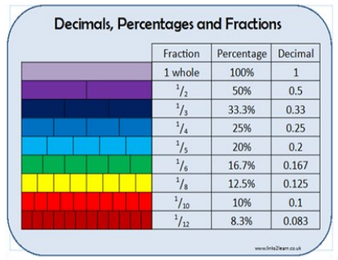
Numerator - The top number of a fraction.

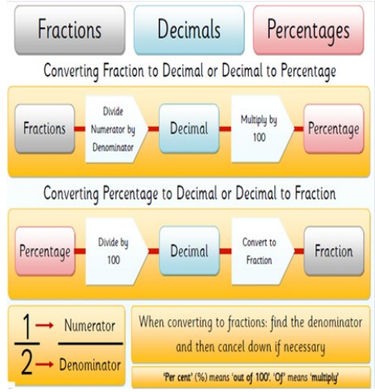
Denominator - The bottom number of a fraction.

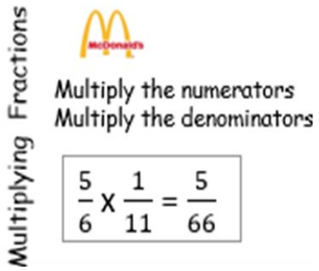
Picture perfect

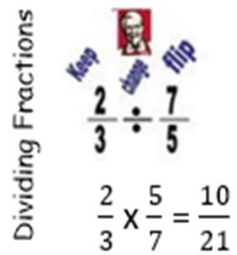


Always remember









Adding and subtracting simple fractions

We can use **equivalent** fractions to add fractions that do not have the same **denominator**.

For example:

$\frac{3}{4} + \frac{1}{8}$

We need to change $\frac{3}{4}$ into an equivalent fraction with a denominator of 8.

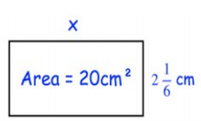
$\frac{3}{4} = \frac{6}{8}$ (×2)

Now we have:

$\frac{6}{8} + \frac{1}{8} = \frac{7}{8}$

Assessment style question

Shown is a rectangle.
Find the value of x

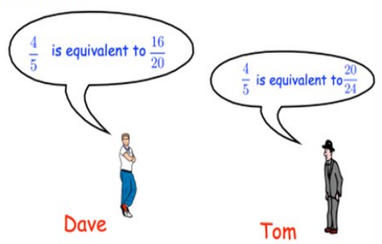


A wall measures $3\frac{3}{4}\,m$ by $4\frac{1}{3}\,m$

Each can of paint cover 2.5m² and costs £5.50

Work out the cost of painting the wall.

Dave and Tom are discussing fractions.
Is either man correct?



Maths - Foundation

Coordinates & Linear Graphs

Key vocabulary

Coordinates - a set of value that show an exact position on a coordinate grid

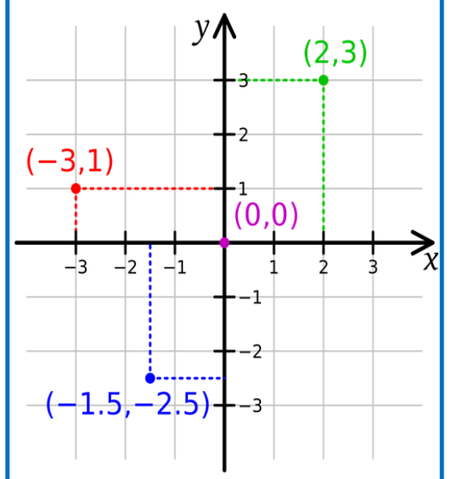
Linear equation - an equation, when plotted, makes a straight line

Gradient - the steepness of the line of a linear equation

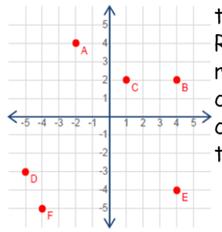
y-intercept - where the linear equation cuts the y-axis

Substitution - when you replace an unknown for a given value

Picture perfect



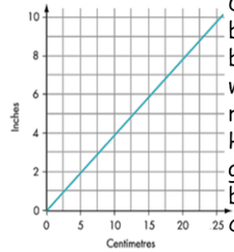
Reading and plotting coordinates



Coordinates are written in the form of (x, y). Remember the rule for reading and plotting coordinates is 'along the corridor and up or down the stairs'.

A = (-2, 4) B = (4, 2)
C = (1, 2) D = (-5, -3)
E = (4, -4) F = (-4, -5)

Using a conversion graph



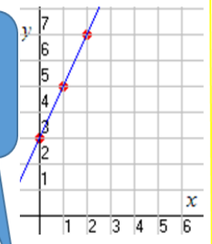
Conversion graphs can be used to convert between any 2 units which have a linear relationship.
Here, you can use the graph to convert between inches and centimetres

Drawing a graph to represent a linear equation

When you are asked to draw the line of a linear equation you follow these steps:

1. Complete a table of values by substituting in the value of x into the equation
2. Write the list of coordinates
3. Plot the coordinates
4. Draw a line through the all the coordinates

This means 2 multiplied the value of x, then add on 3



Example
Complete the table of values for the equation y = 2x + 3 for the values of x from 0 to 2

X	0	1	2
y	2x0+3 =3	2x1+3 =5	2x2+3 =7

Coordinates are (0,3) (1,5) and (2,7)

Assessment style question

Question 1:

(a) Plot the coordinates A (−4, 1), B (1, −2) and C (2, 1)
(b) ABCD is a kite.
(c) Plot D
(d) Write down the coordinates of the point D.

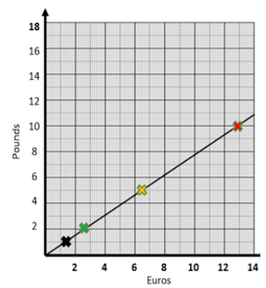
Question 2:

The distance between the points (−3, −4) and (q, 5) is 15. Find the possible values of q.

Drawing a conversion graph

You can plot known conversions on a graph to help you to convert other unknown amounts.

Current exchange rate
£1 = € 1.29
£2 = € 2.58
£5 = € 6.45
£ 10 = € 12.90



Can you use the graph to convert 10 Euros into Pounds?

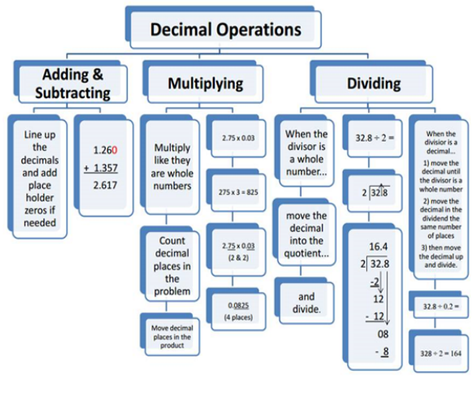
Maths - Foundation

Basic Decimals

Key vocabulary

Integer: A **whole number** that can be positive, negative or zero.
Decimal: A number with a **decimal point** in it. Can be positive or negative.
Decimal Point: a full point or dot placed after the figure representing units in a decimal fraction

Picture perfect



Assessment style question

A grain of rice has a mass of 0.015g
How many grains are there in 300g of rice?

A type of pebble has a mass of 0.8g
How many pebbles are there in 40kg?

Class 8A are going on a trip to a windmill

The trip costs £3.70 each and there are 26 students in 8A.
How much money should be collected?

Mr.Jenkins is building a fence for his garden.
The fence costs £12.60 per metre to build.
The fence is 5.3 metres long.

Work out the total cost of building the fence.

Always remember

To add decimals, follow these steps:
•Write down the numbers, one under the other, with the **decimal points lined up**
•**Put in zeros** so the numbers have the same length
•**Then add** using column addition, remembering to put the decimal point in the answer

Example: Add 1.452 to 1.3

Line the decimals up:	1.452
	+ 1.3
"Pad" with zeros:	1.452
	+ 1.300
Add:	1.452
	+ 1.300
	2.752

To subtract, follow the same method: line up the decimals, then subtract

Example: What is 7.368 – 1.15 ?

Line the decimals up:	7.368
	– 1.15
"Pad" with zeros:	7.368
	– 1.150
Subtract:	7.368
	– 1.150
	6.218

To check we can add the answer to the number subtracted

Putting In Zeros

Why can we put in extra zeros?

A zero is really saying "there is no value at this decimal place".

- In a number like 10, the zero is saying "no ones"
- In a number like 2.50 the zero is saying "no hundredths"

So it is safe to take a number like 2.5 and make it 2.50 or 2.500 etc

But DON'T take 2.5 and make it 20.5, that is plain wrong.

How to Multiply Decimals
Just follow these steps:
•Multiply normally, ignoring the decimal points.
•**Then** put the decimal point in the answer - it will have as many decimal places as the two original numbers combined.
In other words, just count up how many numbers are after the decimal point in *both* numbers you are multiplying, then the answer should have that many numbers after *its* decimal point.

Example: Multiply 0.03 by 1.1

start with:	**0.03 × 1.1**
multiply without decimal points:	**3 × 11 = 33**
0.03 has **2 decimal places**, and 1.1 has **1 decimal place**, so the answer has **3 decimal places**:	**0.033**

Let us multiply the 0.2 by 10, which shifts the decimal point out of the way:

0.2 × 10 = 2

But we must **also** do it to the 15:

15 × 10 = 150

So 15 ÷ 0.2 has become 150 ÷ 2 (they are **both** 10 times larger):

150 ÷ 2 = 75

And so the answer is:

15 ÷ 0.2 = 75

How to Divide Decimals
The trick is to get rid of the decimal point from the number we are dividing by.
How? We can "shift the decimal point" out of the way by multiplying by 10, as many times as we need to.
But we must do the **same thing** to both numbers in the division.

Example above: 15 divided by 0.2

Maths - Foundation

Rounding and Estimating

Key vocabulary

Rounding - to alter a number making it less accurate but easier to use in calculations.

Estimation - A rough calculation using a mathematical method.

Decimal place - Place value after the decimal point.

Integer - A whole number.

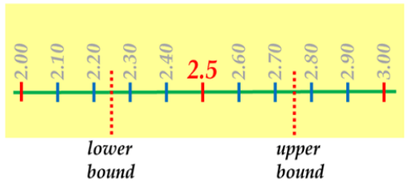
Significant figures (S.F of Sig Fig) - Digits that carry meaning.

Bounds - Upper and lower values of rounded numbers.

Picture perfect

ROUNDING OFF:

If the time is given correct to the nearest half an hour,



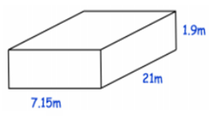
Assessment style question

Nicole has rounded a number to one decimal place.
Her answer is 9.2
Write down 10 different possible numbers that she could have rounded.

In an election 43.8% of people voted for a candidate.
Round this figure to one significant figure

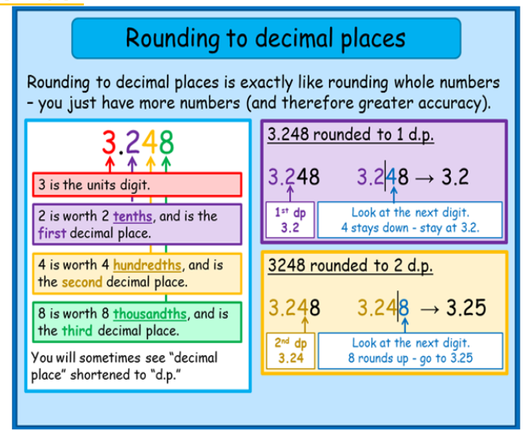
A roll of wallpaper cost £7.85.
Richard buys 29 rolls of wallpaper.
Work out an estimate for the total cost.

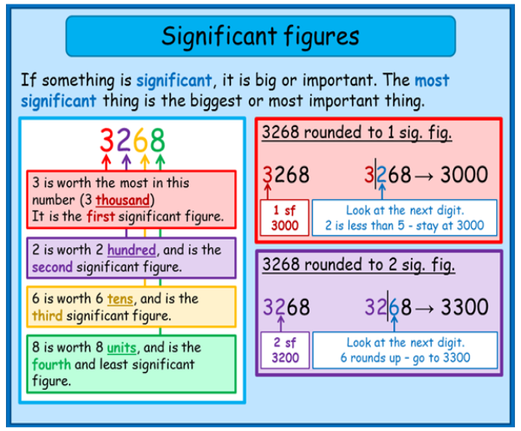
Andrew fills the swimming pool with water at a constant rate of 2.1 litres per second.



Given $1m^3 = 1000$ litres, estimate how long it takes to fill the pool.

Always remember





We estimate by rounding all values to 1 significant figure before completing the calculation.

Maths - Foundation

Frequency Tables

These are a useful and clear way of displaying data

E.g. The table below shows the scores out of ten for 20 students

Mark	Tally	Frequency
4	II	2
5	II	2
6	IIII	4
7	~~IIII~~	5
8	IIII	4
9	II	2
10	I	1

Frequency means how often something occurs

This means 5 students scored 7 marks in their test

Grouped Frequency Tables

These contain sorted data in groups called **classes**

E.g. The table below shows the ages of people taking swimming lessons

Class Interval	Frequency
15 – 25	60
25 – 35	35
35 – 45	22
45 – 55	18
55 – 65	15

Total frequency will tell you the total number of people taking swimming lessons

This means 18 people who took swimming lessons were between the ages of 45 and 55

Classes or class widths

Comparative Bar Charts

The table shows the number of cars sold by Kitty and George in the first four months of 2014.

	January	February	March	April
Kitty	2	5	13	10
George	4	7	9	10

The chart has a key to make it easier to understand.

A **comparative bar chart** allows you to easily compare the number of cars Kitty and George sold each month.

Charts

Two-Way Tables

These are used to show how data falls into 2 different categories

For example gender and favourite sport to watch

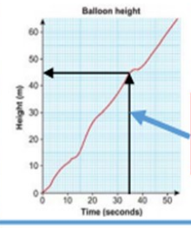
What is your favorite sport to watch on television?			
	Football	Basketball	Baseball
Males	40	22	15
Females	12	16	45
Total	52	38	60

A two-way table divides data into groups in rows going across and columns going down the table

Time-Series Graph

These are used to show how something changes over time. It is a line graph with time plotted along the horizontal axis

For example the height of a balloon at different times



You can estimate the height of the balloon at different times using the graph

E.g. the height of the balloon at 35 seconds is approximately 45m as shown by the arrows on the graph

Stem and Leaf Diagrams

This shows numerical data split into a 'stem' and 'leaves'. The leaf is usually the last digit and the stem is the other digits.

Here are the heights of some students (in cm).
169, 163, 153, 173, 166, 178, 177
Construct a stem and leaf diagram for this data.

15 | 3
16 | 9 3 6
17 | 3 8 7

Decide on a stem. Write the numbers in your diagram as you work along the data list.

15 | 3
16 | 3 6 9
17 | 3 7 8

Put the leaves in your diagram in order.

Key 15|3 means 153 cm — Write a key for your diagram.

A back-to-back stem and leaf diagram compares 2 sets of data. E.g. the ages of males and females

Females		Males
8 5	5	6 7 9
7 5 4 3	6	8 3 5
9 8 6 1	7	9 2 3 5 7 8
	8	
	9	
	10	1 3 7

5|7 means 75 — 10|1 means 101

Pie Charts

This is a circle divided into **sectors.** Each sector represents a set of data. Pie charts are excellent for displaying the most/ least popular type of something.

Plotting pie charts example

The table show the match results of a football team.

Result	Won	Drawn	Lost
Frequency	28	12	20

28 + 12 + 20 = 60 ← The total number of games is the total frequency.

1 game = 360° ÷ 60 games = 6° per game ← Work out the angle for **one** game.

360° in a circle

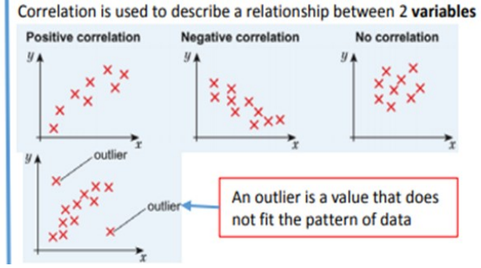
28 games won = 28 × 6° = 168°
12 games drawn = 12 × 6° = 72°
20 games lost = 20 × 6° = 120°
← Work out the angle for each result.

Draw the pie chart. Give it a title and a key. Or label each section

Scatter Graphs

A scatter graph allows you to see the **relationship** between 2 sets of data. E.g. your height and your stride length.

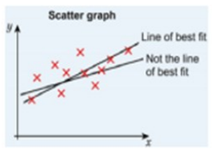
Correlation is used to describe a relationship between 2 **variables**



An outlier is a value that does not fit the pattern of data

A line of best fit

This is a straight line drawn through the middle of the points on a scatter graph. It should pass as near as many points as possible and represents the **trend** of the points.



A line of best fit can be used to predict data values within the range of data given.
This is called **interpolation**
It can also be used to predict data values outside the range of data given.
This is called **extrapolation.**

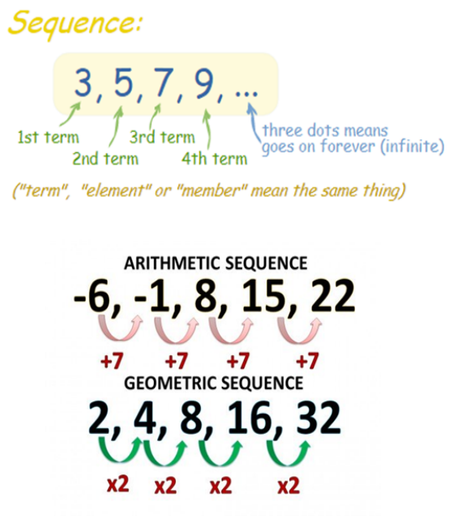
Maths - Foundation

Sequences

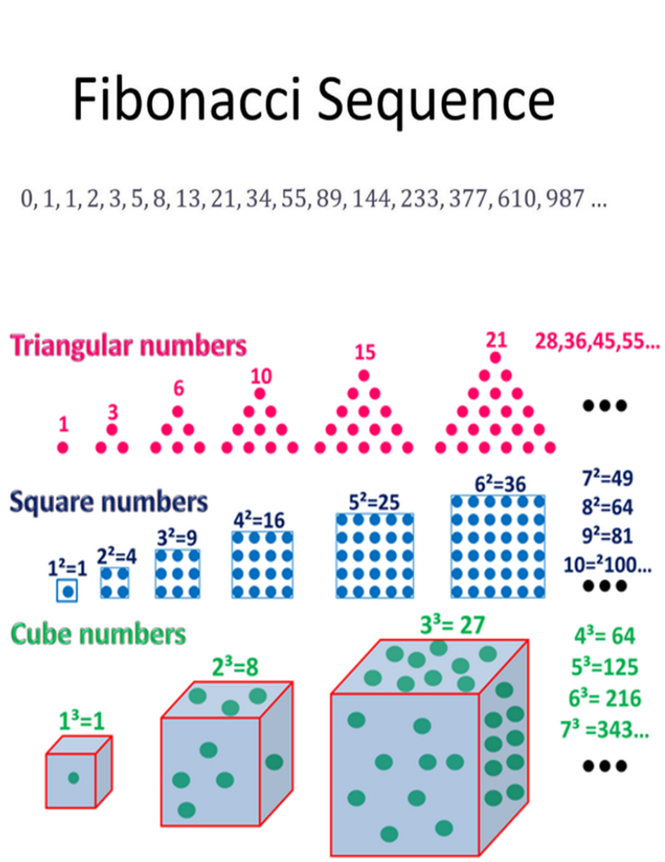
Key vocabulary

Sequence
Term
Rule
Term to term rule
Nth term
Arithmetic sequence
Geometric sequence
Triangular numbers
Fibonacci sequence

Picture perfect



Always remember



Assessment style question

The nth term of a sequence is $2n^2$

(i) Find the 4th term of the sequence.

.....

(ii) Is the number 400 a term of the sequence?

.....

Give reasons for your answer.

.....

.....

The first five terms of an arithmetic sequence are

2 7 12 17 22

Write down, in terms of n, an expression for the nth term of this sequence.

Maths - Higher

Angles and Scale

Key vocabulary

Angle, degree, calculate, parallel, polygon, perpendicular, isosceles, corresponding, alternate, co-interior, vertically opposite, bearing, scale, three figure bearing, north line

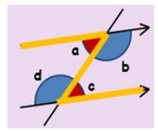
Assessment style question:

The diagram shows a triangle and a trapezium.

Not drawn accurately

Prove that $a = b$

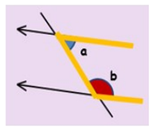
Picture perfect



Alternate angles are equal

Angle a = c and b = d

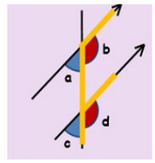
They make a Z shape



Supplementary angles add up to 180°

Angle a + b = 180°

They make a C shape



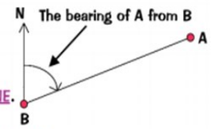
Corresponding angles are equal

Angle a = c and b = d

They make an F shape

Bearings

To find or plot a bearing you must remember the three key words:

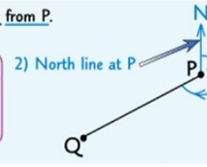


1) 'FROM' Find the word 'FROM' in the question, and put your pencil on the diagram at the point you are going 'from'.

2) NORTH LINE At the point you are going FROM, draw in a NORTH LINE. (There'll often be one drawn for you in exam questions.)

3) CLOCKWISE Now draw in the angle CLOCKWISE from the north line to the line joining the two points. This angle is the required bearing.

Find the bearing of Q from P.



1) 'From P'

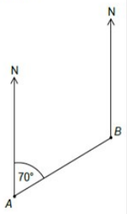
2) North line at P

3) Clockwise, from the N-line. This angle is the bearing of Q from P. Measure it with your protractor — 245°.

ALL BEARINGS SHOULD BE GIVEN AS 3 FIGURES
e.g. 176°, 034° (not 34°), 005° (not 5°), 018° etc.

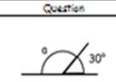
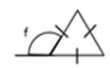
Angles

The bearing of *B* from *A* is 070°



Circle the bearing of *A* from *B*.

Always remember

Question	Answer	Reason
	150°	Angles on a straight line add up to 180°
	54°	Vertically opposite angles are equal
	150°	Angles around a point add up to 360°
	28°	Angles in a triangle add up to 180°
	40°	Angles in a triangle add up to 180°
	120°	Equilateral triangles have all equal angles and angles in a triangle add up to 180° and angles on a straight line add up to 180°
	58°	Angles on a straight line add up to 180° and angles in a triangle add up to 180°
	111°	Isosceles triangles have two equal angles and angles in a triangle add up to 180° and angles on a straight line add up to 180°

Maths - Higher

Basic Number

Key vocabulary

Multiple - a number which can be divided by another number without a remainder.
Divisor – the number you divide by.
Factor - An exact divisor of a number.
Prime - a number which only has two factors
Product – the result of multiplying to numbers together
Prime Factorisation – a list of prime numbers which multiply together to create another number

Picture perfect

Prime Factor Decomposition
Break numbers down into prime factors
Tree method and ladder method

72 = 2 × 2 × 2 × 3 × 3
$72 = 2^3 \times 3^2$

84 = 2 × 2 × 3 × 7
$84 = 2^2 \times 3 \times 7$

Divide by a prime
Multiply Primes
Write in index form
Prime factor decomposition

HCF
Highest common factor
The largest number that divides into two or more numbers
Use long format of Prime Factor Decomposition

HCF of 48 and 120
48 = 2 × 2 × 2 × 2 × 3
120 = 2 × 2 × 2 × 3 × 5
2×2×3 = 12

HCF of 84 and 980
84 = 2 × 2 × 3 × 7
980 = 2 × 2 × 5 × 7 × 7
2×2×7 = 28

Identify shared factors
Multiply values

LCM
Lowest common multiple
The smallest number that occurs in the times table of two or more numbers.

LCM of 6 and 45
6 = 2×3
45 = 3×3×5
2×3×3×5 = 90

LCM of 48 and 180
48 = 2^4×3
180 = 2^2×3^2×5
2^4×3^2×5 = 720

Multiply together all prime factors apart from duplicates
In index form: Multiply Highest Power of each prime

Always remember

Primes

A **prime** number is a number which only has two factors – itself and 1.

The first 10 **prime** numbers are:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29

A common mistake is that people think 1 is a **prime** number. The only way to make 1 is by doing 1 x 1 , so the number 1 only has one factor NOT two factors, therefore it is NOT **prime**.

Examples
Is 11 **prime**? - 1 x 11 Yes it's **prime**
Is 12 **prime**? - 1 x 12, 2 x 6, 3 x 4 No it's not **prime**

Factors

An exact divisor of a number is called a **factor**.

Factors can be written in pairs to help find all the factors of a given number.

E.g. **Factors** of 12 can be written like this…..

Factors of 12

1, 2, 3, 4, 6, 12

Assessment style question

A bus leaves Antrim Bus Station every 12 minutes.
A train leaves Antrim Train Station every 18 minutes.
At 8am a bus and a train leave the stations at the same time.

(a) When is the next time that a bus and a train leave at the same time?
(b) Between 8am and 11am, on how many occasions does a bus and a train leave at the same time?

The lowest common multiple of two numbers is 60.
Only one of the numbers is a multiple of 4.
Write down two possible numbers.

Penny and Kenny have the same number of football cards.
Penny has sorted her cards into piles of 10.
Kenny has sorted his cards into piles of 18.
Penny has less than 100 cards.
How many football cards do they have?

Olivia thinks of two numbers.
The lowest common multiple (LCM) of the two numbers is 36.
The highest common factor (HCF) of the two numbers is 3.
Both numbers are less than 15.
Write down two possible numbers that Olivia could be thinking of.

Multiples

A **multiple** is a number which can be divided by another number without a remainder.

For example: 15, 20 100, 150 are **multiples** of 5 because they can be divided by 5 and have no remainder

Maths - Higher

Basic Algebra

Key vocabulary

Powers, Indices, Reciprocal, Roots, Brackets, Multiply out, Expand, Simplify, Factor, Factorise, Algebra, Notation, Coefficient, Expression, Equation, Formula, Identity, Inequality, Term, Collecting like terms,

Picture perfect

Expanding brackets:

- Expanding means remove brackets
- Use the claw

e.g. $3(5a-2) = (3\times 5a) - (3\times 2)$

$= 15a - 6$

Factorising expressions:

- The opposite (inverse) of expanding
- Answer will include brackets
- Look for common factors (numbers and algebra)
- Always choose the HCF

e.g. 1 $10a + 15 = 5(2a+3)$

10 & 15 both in the 5 times table

$10a = 5\times 2a$ $15 = 5\times 3$

Always remember

Algebraic Notation

Algebra uses a shorthand system where variables (symbols) are represented by letters.

E.g. $a + a = 2a$

$2 \times b = 2b$

$c \times c = c^2$

$3d \times 2e = 6de$

Factorising Brackets

To factorise into one bracket, find the highest common factor for each term.

E.g. $4x + 10 = 2(2x + 5)$

Expanding Brackets

To expand one bracket:
make sure the term on the outside multiplies everything on the inside.

E.g. $4(2x - 3) = 8x - 12$

Simplifying expressions (adding/subtracting)

- Can only simplify like terms.
- Be sure to include the sign before the term

e.g. $2a + 3b - a + 4b = a + 7b$

$2a - a = a$ $+3b + 4b = +7b$

Term = 3a
Expression = 5x + 2
Equation = 5x – 3 = 47
Formula = C = πd
Identity= $10x + 4 \equiv 2(5x + 2)$

Assessment style question: Circle the correct words to complete each sentence:

(a) $x^2 + 3x = 16$ is	an expression	an equation	a formula	an identity
(b) $x^2 + 3x = 16$ is	an expression	an equation	a formula	an identity
(c) $(x + 3)^2 \equiv x^2 + 6x + 9$ is	an expression	an equation	a formula	an identity

$(3x + 1)(x - 2) + ax + b \equiv 3x^2 + 8x - 5$

Work out the values of a and b.

Maths - Higher

Decimals and fractions

Key vocabulary

Mixed number

Equivalent fraction

Simplify, cancel

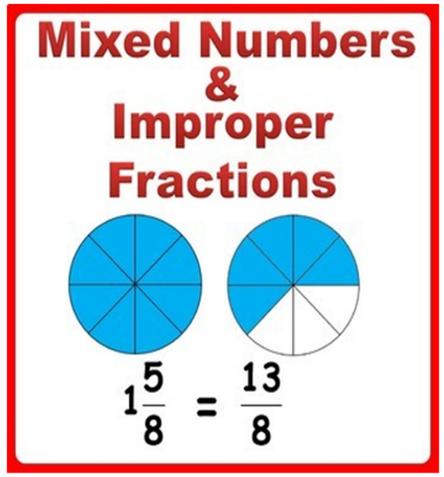
Lowest terms

Proper fraction, improper fraction, top-heavy fraction, vulgar fraction

Numerator, denominator

Percent, percentage

Picture perfect



Assessment style question

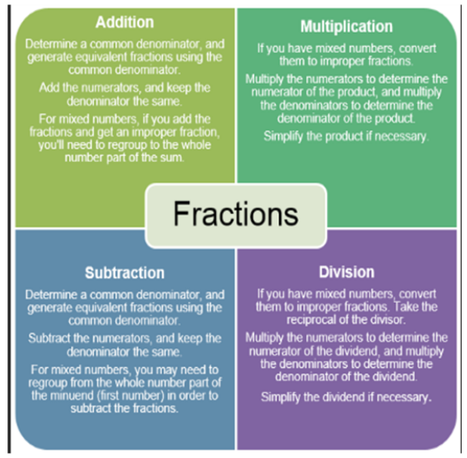
Work out $4\frac{2}{3} - 1\frac{4}{5}$

Give your answer as a mixed number.

Is Sophia correct?
Explain your answer.

(1)

Always remember



Multiplying decimals	Dividing decimals
0.3 x 1.5 Remove the decimal points by multiplying by powers of 10. 3 x 15 = 45 Replace the total number of decimal places from the question. 45 = 0.45	1.5 ÷ 0.05 Remove the decimal points by multiplying by the same power of 10. 1.5 ÷ 0.05 x100 x100 150 ÷ 5 Answer = 3 We do not need to replace the decimal point as we have multiplied each part by the same power of 10.

Maths - Higher

Linear graphs

Key vocabulary

Co-ordinates
Quadrant
Origin
Equation
Horizontal
Vertical
Parallel
Intersect
Gradient
Plot
Sketch
Linear

Picture perfect

All straight-line graphs have an equation that is written that is written like this:

$$y = mx + c$$

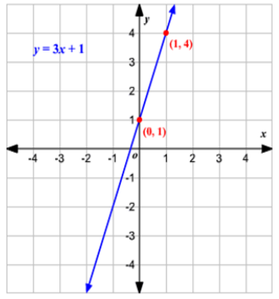
Y gives you the y-coordinate we need to plot the graph

M is the **gradient** of the line, the 'steepness' of it.

X is the x-coordinate we need to plot the graph

C is called the **y-intercept.** It's the point where the line crosses the y-axis.

Drawing Graphs
Step 1 – Draw a table up
Step 2 – Plot co-ordinates on axes
Step 3 - Join the points

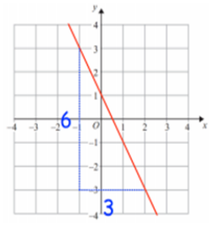


Assessment style question

(a) Draw $y = x + 1$ and $y = 2x - 1$ on the same set of axes.

(b) Where do the two graphs intersect?

Alisha says that the gradient of the line is 2. Explain her mistake.



Always remember

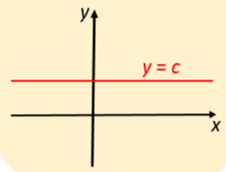
A line sloping upwards from left to right has a **positive** gradient.		m is positive
A line sloping downwards from left to right has a **negative** gradient.		m is negative
Parallel lines have the **same** gradient.		$m_1 = m_2$
A **horizontal** line has a gradient of 0.		m = 0
The gradient of a **vertical** line is undefined.		m is undefined

Coordinates are given in the form (x, y). The first number is the position in the x-direction (horizontal) and the second is the position in the y-direction (vertical).

$$\text{Gradient} = \frac{\text{change up}}{\text{change right}} \text{ or } \frac{\text{change in } y}{\text{change in } x}$$

Graph Horizontal and Vertical Lines

A horizontal line has an equation of the form $y = c$. It has a slope of 0.



A vertical line has an equation of the form $x = a$. It has a an undefined slope.

Maths - Higher

Rounding and Estimating

Key vocabulary

Rounding – to alter a number making it less accurate but easier to use in calculations.

Estimation – A rough calculation using a mathematical method.

Decimal place – Place value after the decimal point.

Integer – A whole number.

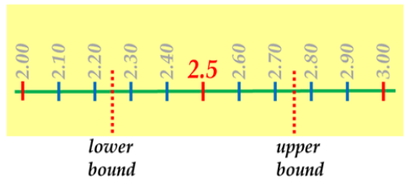
Significant figures (S.F of Sig Fig) – Digits that carry meaning.

Bounds – Upper and lower values of rounded numbers.

Picture perfect

ROUNDING OFF:

If the time is given correct to the nearest half an hour,

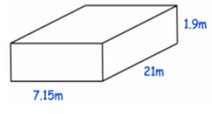


Assessment style question

Nicole has rounded a number to one decimal place.
Her answer is 9.2
Write down 10 different possible numbers that she could have rounded.

A roll of wallpaper cost £7.85.
Richard buys 29 rolls of wallpaper.
Work out an estimate for the total cost.

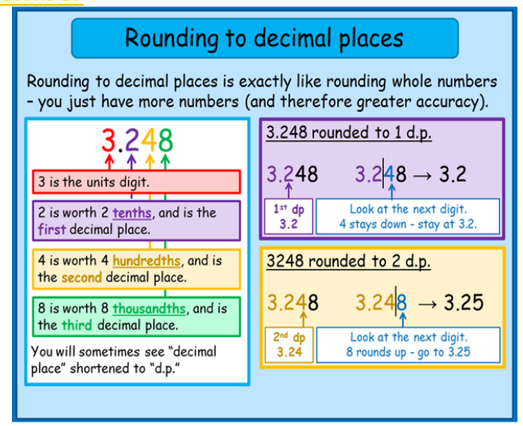
In an election 43.8% of people voted for a candidate.
Round this figure to one significant figure

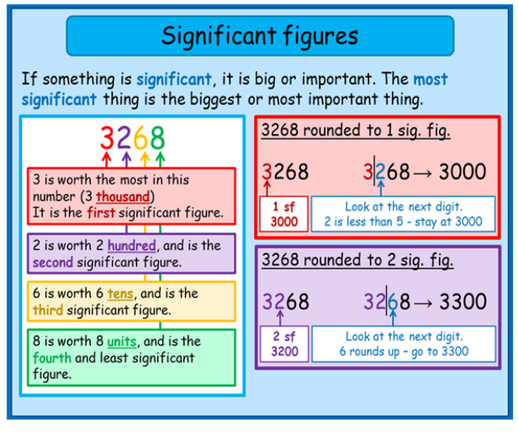


Andrew fills the swimming pool with water at a constant rate of 2.1 litres per second.

Given $1m^3 = 1000$ litres, estimate how long it takes to fill the pool.

Always remember





We estimate by rounding all values to 1 significant figure before completing the calculation.

Maths - Higher

Frequency Tables

These are a useful and clear way of displaying data

E.g. The table below shows the scores out of ten for 20 students

Mark	Tally	Frequency
4	II	2
5	II	2
6	IIII	4
7	𝍸	5
8	IIII	4
9	II	2
10	I	1

Frequency means how often something occurs

This means 5 students scored 7 marks in their test

Grouped Frequency Tables

These contain sorted data in groups called **classes**

E.g. The table below shows the ages of people taking swimming lessons

Class Interval	Frequency
15 – 25	60
25 – 35	35
35 – 45	22
45 – 55	18
55 – 65	15

Total frequency will tell you the total number of people taking swimming lessons

This means 18 people who took swimming lessons were between the ages of 45 and 55

Classes or class widths

Comparative Bar Charts

The table shows the number of cars sold by Kitty and George in the first four months of 2014.

	January	February	March	April
Kitty	2	5	13	10
George	4	7	9	10

The chart has a key to make it easier to understand.

A **comparative bar chart** allows you to easily compare the number of cars Kitty and George sold each month.

Charts

Two-Way Tables

These are used to show how data falls into 2 different categories

For example gender and favourite sport to watch

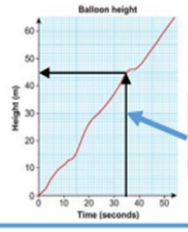
What is your favorite sport to watch on television?			
	Football	Basketball	Baseball
Males	40	22	15
Females	12	16	45
Total	52	38	60

A two-way table divides data into groups in rows going across and columns going down the table

Time-Series Graph

These are used to show how something changes over time. It is a line graph with time plotted along the horizontal axis

For example the height of a balloon at different times



You can estimate the height of the balloon at different times using the graph

E.g. the height of the balloon at 35 seconds is approximately 45m as shown by the arrows on the graph

Stem and Leaf Diagrams

This shows numerical data split into a 'stem' and 'leaves'. The leaf is usually the last digit and the stem is the other digits.

Here are the heights of some students (in cm).
169, 163, 153, 173, 166, 178, 177
Construct a stem and leaf diagram for this data.

15 | 3
16 | 9 3 6 — Decide on a stem. Write the numbers in your diagram as you work along the data list.
17 | 8 8 7

15 | 3
16 | 3 6 9 — Put the leaves in your diagram in order.
17 | 7 8

Key 15 | 3 means 153 cm — Write a key for your diagram.

A back-to-back stem and leaf diagram compares 2 sets of data. E.g. the ages of males and females

Females		Males
8 5 7	6	7 9
7 5 4 3 0	7	8 3 5
9 8 6 1	8	9 2 3 5 7 8
	9	1 3 7
	10	

5 | 7 means 75

10 | 1 means 101

Pie Charts

This is a circle divided into **sectors.** Each sector represents a set of data. Pie charts are excellent for displaying the most/ least popular type of something.

Plotting pie charts example

The table show the match results of a football team.

Result	Won	Drawn	Lost
Frequency	28	12	20

28 + 12 + 20 = 60 — The total number of games is the total frequency.

1 game = 360° ÷ 60 games = 6° per game — Work out the angle for **one** game.

360° in a circle

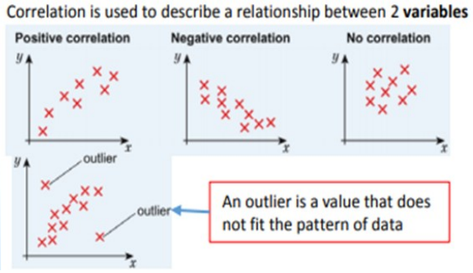
28 games won = 28 × 6° = 168°
12 games drawn = 12 × 6° = 72°
20 games lost = 20 × 6° = 120° — Work out the angle for each result.

Draw the pie chart. Give it a title and a key. Or label each section

Scatter Graphs

A scatter graph allows you to see the **relationship** between 2 sets of data. E.g. your height and your stride length.

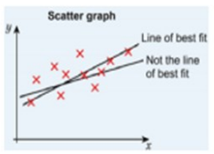
Correlation is used to describe a relationship between 2 **variables**



An outlier is a value that does not fit the pattern of data

A line of best fit

This is a straight line drawn through the middle of the points on a scatter graph. It should pass as near as many points as possible and represents the **trend** of the points.



A line of best fit can be used to predict data values within the range of data given.
This is called **interpolation**
It can also be used to predict data values outside the range of data given.
This is called **extrapolation.**

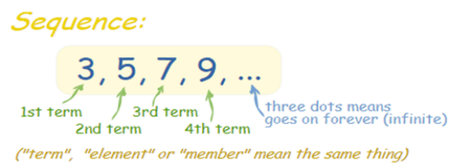
Maths - Higher

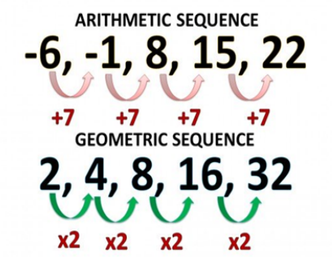
Sequences

Key vocabulary

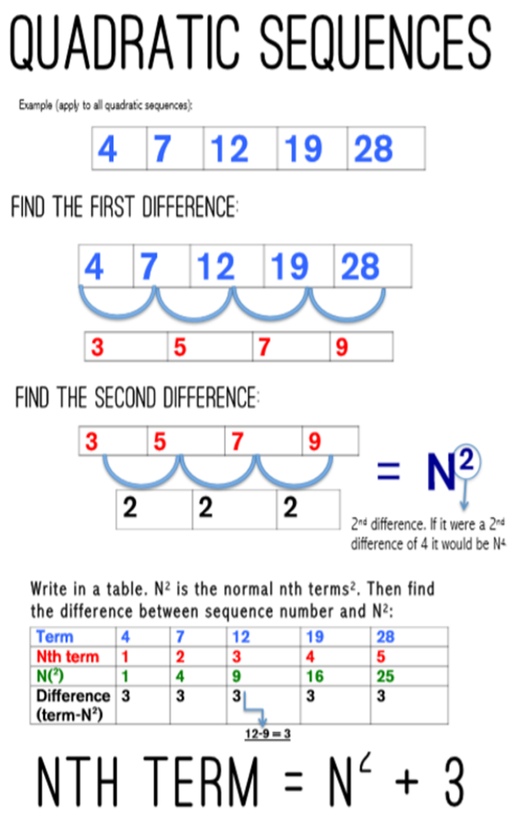
Sequence
Term
Rule
Term to term rule
Nth term
Arithmetic sequence
Geometric sequence
Triangular numbers
Fibonacci sequence

Picture perfect





Always remember



Assessment style question

The nth term of a sequence is $2n^2$

(i) Find the 4th term of the sequence.

.....

(ii) Is the number 400 a term of the sequence?

.....

Give reasons for your answer.

.....

.....

The first five terms of an arithmetic sequence are

2 7 12 17 22

Write down, in terms of n, an expression for the nth term of this sequence.