

Maths - Foundation

Basic percentages

Key vocabulary

Fraction - A quantity which is not a whole number.

Decimal - A decimal number is often used to mean a number that uses a decimal point followed by digits that show a value smaller than one.

Percentage - Amount out of one hundred.

Increase - To make bigger.

Decrease - To make smaller.

Depreciate - Decrease in value over time.

Multipliers - a quantity by which a given number is to be multiplied.

Assessment style question

A primary school has 212 students.
50% of the students are boys.
How many of the students are boys?

A fish tank, that is full of water, has sprung a leak.
12% of the water is lost every hour.
What percentage of the water is lost after three hours?

A cereal bar weighs 24g.
The cereal bar contains 3.8g of protein.
Work out what percentage of the cereal bar is protein.

Picture perfect

Reverse percentages

John pays £60 for a bag after getting 20% discount. How much did it originally cost?

Remember: Original price is always equal to 100%

Sale price = 100% - 20% = 80%

$\div 80$ → 80% = £60 → $\div 80$
 1% = 0.75
 $\times 100$ → 100% = £75 → $\times 100$

Always remember

Decimals, Percentages and Fractions			
Fraction	Percentage	Decimal	
1 whole	100%	1	
$\frac{1}{2}$	50%	0.5	
$\frac{1}{3}$	33.3%	0.33	
$\frac{1}{4}$	25%	0.25	
$\frac{1}{5}$	20%	0.2	
$\frac{1}{6}$	16.7%	0.167	
$\frac{1}{8}$	12.5%	0.125	
$\frac{1}{10}$	10%	0.1	
$\frac{1}{11}$	9.1%	0.091	

on a calculator

39% of 82
 0.39×82 → Change to a decimal and multiply

fraction to %

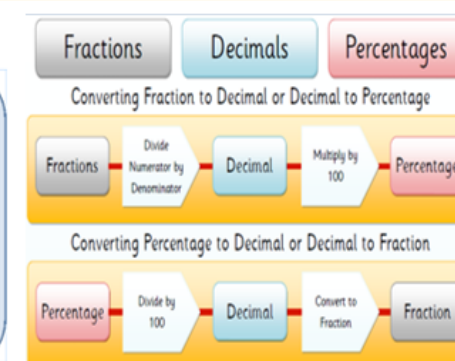
$\frac{15}{20} = \frac{75}{100} = 75\%$
 OR
 $15 \div 20 \times 100 = 75\%$

without a calculator

50% - half
 25% - half and half
 75% - 50% + 25%
 10% - divide by 10
 5% - half 10%
 20% - double 10%

Simple interest = amount \times multiplier \times time

Compound interest = amount \times multiplier^{time}



increasing

Increase £60 by 12%

12% of 60 = $0.12 \times 60 = £7.20$

New amount = £60 + £7.20 = £67.20

ADD

decreasing

decrease £60 by 12%

12% of 60 = $0.12 \times 60 = £7.20$

New amount = £60 - £7.20 = £52.80

SUBTRACT

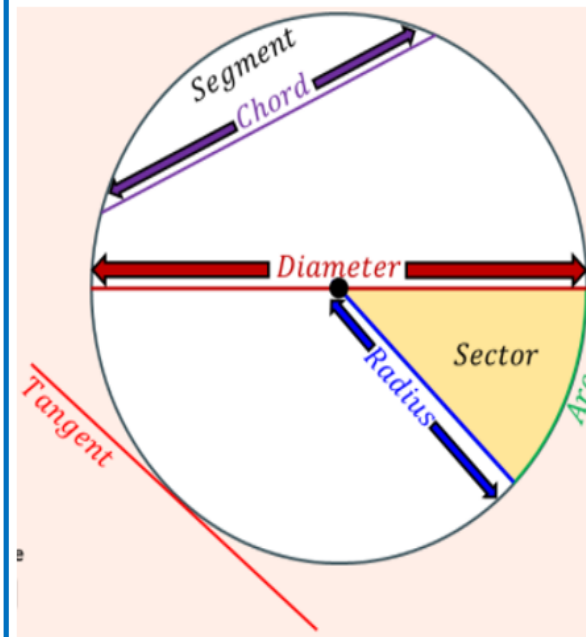
Maths - Foundation

Circumference and area

Key vocabulary

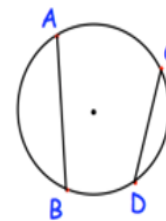
Circumference
The perimeter around the circle
Diameter
The distance across the centre of the circle
Radius
The distance from the centre to the edge of the circle
Sector
Part of the area of a circle, enclosed by two radii
Arc
Part of the circumference of a circle
Tangent
A straight line that touches the curve of the circle at a point
Chord
A straight line segment between two points on the circle edge
Segment
The area created by the chord

Picture perfect



Assessment style question

Nicole is a wedding organiser.
The guests are to sit at circular tables with a diameter of 180cm.
Each guest needs 70cm around the circumference of the table.
There are 18 tables at the venue.
A total of 145 guests are attending the wedding
Are there enough tables?



- Draw a circle with two chords, AB and CD.
- Construct the perpendicular bisector of AB.
- Construct the perpendicular bisector of CD.
- What do you notice about where the two perpendicular bisectors meet?

Always remember

Area

$A = \pi r^2$ → Pi times the radius squared

Diameter is double the radius

$A = \pi \times 6.5^2$
 $A = \pi \times 42.25$
 $A = 132.73m^2$

Sector

Calculate the **proportion** of the circle required then use area formula

$A = \frac{n^\circ}{360} \pi r^2$

$\frac{85^\circ}{360^\circ} (\pi \times 6^2)$
 $26.7cm^2$

Circumference

$C = \pi d$
 $C = 2\pi r$

The circumference is always about three times the length of the diameter

$C = \pi \times 12cm$
 $C = 37.7cm$

Arc length

Calculate the **proportion** of the circle required then use circumference formula

$L = \frac{n^\circ}{360} \pi d$

$\frac{85^\circ}{360^\circ} (\pi \times 12)$
 $8.90cm$

Maths - Foundation

Solving equations

Key vocabulary

Equation
Expression
Identity
Formulae
Inequality
Solve
Simplify
Like terms
Co-efficient
Expand
Factorise

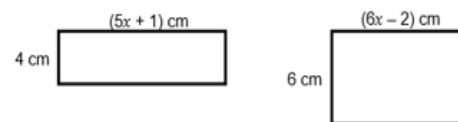
Picture perfect

$$\begin{array}{rcl}
 5x - 2 & = & 3x + 4 \\
 -3x & & -3x \\
 \hline
 2x - 2 & = & 4 \\
 +2 & & +2 \\
 \hline
 2x & = & 6 \\
 & & x = 3
 \end{array}$$

Assessment style question

(a) Solve $4x + 1 = 9$

These two rectangles have the same area.



(b) Solve $2x - 5 = 4$

Write down an equation to show this.

[1 mark]

Always remember

Solving simple two-step equations

To solve an equation, find the value that makes the equation true.

Solve $2x + 3 = 13$

This means: $x \times 2 + 3 = 13$

To solve, we reverse the process:

$$\begin{array}{rcl}
 x \times 2 + 3 & = & 13 \\
 x \times 2 & = & 10 \\
 2x & = & 10 \\
 x & = & 5
 \end{array}$$

Use the opposite (inverse) operation and undo in reverse order.

We have solved the equation when we get to a single value of x (here, $x = 5$).

Solve $4x + 6 = 14$

$$\begin{array}{rcl}
 4x + 6 & = & 14 \\
 4x & = & 8 \\
 x & = & 2
 \end{array}$$

Solve $3x - 8 = 19$

$$\begin{array}{rcl}
 3x - 8 & = & 19 \\
 3x & = & 27 \\
 x & = & 9
 \end{array}$$

Inequality Symbols

\neq	not equal
$<$	less than
\leq	less than or equal to
$>$	greater than
\geq	greater than or equal to

Maths - Foundation

Indices

Key vocabulary

Square: A square number is the result of multiplying a number by itself.

Cube: A cube number is the result of multiplying a number by itself twice.

Root: A root is the reverse of a power.

Indices: These are the squares, cubes and powers.

Operation: In maths these are the functions $\square \square + -$.

Picture perfect

indices

$$a^0 = 1 \quad a^{m/n} = (\sqrt[n]{a})^m$$

$$a^{-n} = \frac{1}{a^n} \quad a^{-1} = \frac{1}{a}$$

$$a^{1/n} = \sqrt[n]{a} \quad a^{1/2} = \sqrt{a}$$

Assessment style question

Question 1: Can you spot any mistakes?

$$6^2 = 12$$

$$1^7 = 7$$

$$2^6 \times 2^3 = 4^9$$

$$7^{15} \div 7^5 = 7^3$$

$$10^4 = 40$$

$$2^6 = 32$$

$$6^3 \times 6^4 = 6^{12}$$

Question 1: Can you spot any mistakes?

Always remember

Basic Laws of Indices

Special indices to consider

$x^1 = x$	Anything to the power 1 = itself
$x^0 = 1$	Anything to the power 0 = 1
$1^x = 1$	1 to the power of anything = 1

These laws can be applied if the bases are the same

$$x^a \times x^b = x^{a+b}$$

$$z^3 \times z^7 = z^{10}$$

When multiplying powers with the same base – Add the powers

$$x^a \div x^b = x^{a-b}$$

$$s^2 \div s^5 = s^{-3}$$

When dividing powers with the same base – Subtract the powers

$$(x^a)^b = x^{a \times b}$$

$$(e^4)^3 = e^{12}$$

When raising the power (brackets) – Multiply the powers

Advanced Laws of Indices

Negative Indices

$$x^{-n} \rightarrow \frac{1}{x^n}$$

Find Reciprocal

Apply Positive Power

Apply top and bottom

$$z^{-3} = \left(\frac{1}{z}\right)^3 = \frac{1}{z^3}$$

$$6^{-2} = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

Fractional Indices

$$x^{\frac{m}{n}} = (\sqrt[n]{x})^m$$

Root by denominator first

Then power of numerator

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

$$x^{\frac{1}{4}} = \sqrt[4]{x}$$

$$x^{\frac{2}{3}} = (\sqrt[3]{x})^2$$

$$64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = (4)^2 = 16$$

Negative Fractional Indices

$$x^{-\frac{a}{b}} = \frac{1}{(\sqrt[b]{x})^a}$$

Negative Fractional Powers:
Apply reciprocal first!

$$9^{-\frac{3}{2}} = \frac{1}{9^{\frac{3}{2}}} = \frac{1}{(\sqrt{9})^3} = \frac{1}{(3)^3}$$

Maths - Foundation

Perimeter & Area

Key vocabulary

Perimeter
 Area
 Length
 Width
 Height
 Circumference
 Radius
 Diameter
 Pi (π)
 Units²
 Compound area

Picture perfect



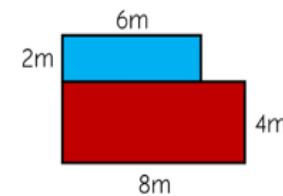
Finding the Area (compound shapes)

To work out the area of compound shapes, you need to break them into individual rectangles.

$$6\text{cm} \times 2\text{cm} = 12\text{cm}^2$$

$$8\text{cm} \times 4\text{cm} = 32\text{cm}^2$$

$$12\text{cm}^2 + 32\text{cm}^2 = 44\text{cm}^2$$



Assessment style question

A rectangle has a perimeter of 18cm.
Write down a possible pair of values for its length and width

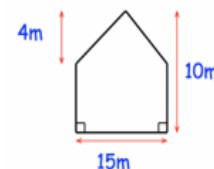
An isosceles triangle has a perimeter of 73cm
An equilateral triangle has a perimeter of 51cm
The triangles are put together to make a kite.



Work out the perimeter of the kite.

Find the area of the triangle with a base of 12cm and perpendicular height of 9cm.

William is painting the side of his house.
He has 8 litres of paint and each litre of paint covers 16m²
Does William have enough paint?



Always remember

Perimeter units include mm, cm, m, km etc.

Area units include mm², cm², m² etc.

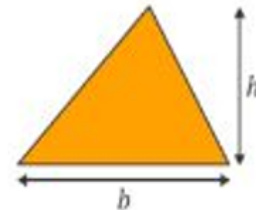
Area of a rectangle:

$$A = l \times w$$



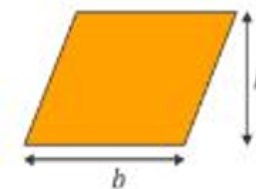
Area of a triangle:

$$A = \frac{bh}{2}$$



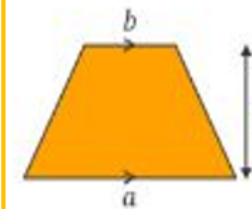
Area of a parallelogram:

$$A = b \times h$$



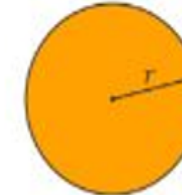
Area of a trapezium:

$$A = \frac{1}{2}(a + b)h$$



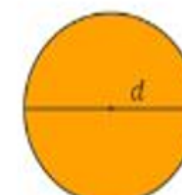
Area of a circle:

$$A = \pi r^2$$



Circumference of a circle:

$$C = \pi d$$



Maths - Foundation

Properties of polygons

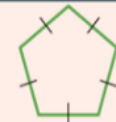
Key vocabulary

Picture perfect

Regular

Side lengths are the same

Interior and exterior angles the same



Irregular

Side lengths are not ALL the same

Interior and exterior angles the not ALL the same



Equal sides are marked with a dash through the line

Convex

The shape is 'bulging' outwards

All angles less than 180°



Concave

The shape has 'caved' inwards

One or more angles is greater than 180°

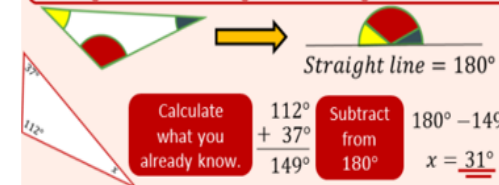


Assessment style question

Always remember

Triangles

All three angles can be orientated to fit on a straight line \rightarrow All angles in a triangle make 180°



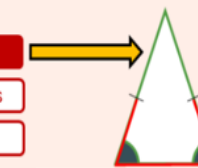
Exterior angle = Sum of two angles on opposite side.



Isosceles triangle

It has two equal lengths

It has two equal angles



They are classified by the number of sides they have

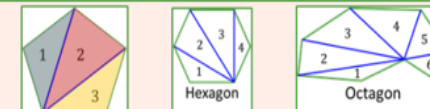
Number of sides	Name of shape
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Polygons

Knowledge of triangles is important

Number of sides	Number of	Sum of interior angles	Regular interior angle	Regular exterior angle
3	1	180°	60°	120°
4	2	360°	90°	90°
5	3	540°	108°	72°
6	4	720°	120°	60°
7	5	900°	129°	51°
8	6	1080°	135°	45°
n	$(n - 2)$	$(n - 2) \times 180^\circ$	$\frac{(n - 2) \times 180^\circ}{n}$	$360^\circ \div n$

The number of triangles in a shape will always be **TWO** less than the number of sides



Maths - Foundation

Ratio

Key vocabulary

Ratio -Ratio compares the size of **one part** to **another part**.

Proportion -Proportion compares the size of **one part** to the size of the **whole**.

Proportional - a change in one is always accompanied by a change in the other.

Simplifying - Divide each part of the ratio by a common factor

Equivalent- Ratios are equivalent if they have the same simplest form.

Picture perfect

Share £20 in the ratio **2:5:3**

1) Find the **total number of parts**

$$2 + 5 + 3 = 10$$

2) Divide the **amount** by the **total number of parts**

$$£20 \div 10 = £2 = 1 \text{ part}$$

3) Multiply each number in the **ratio** by the value of **1 part**

$$\begin{array}{ccc} 2 & : & 5 & : & 3 \\ \downarrow \times £2 & & \downarrow \times £2 & & \downarrow \times £2 \\ £4 & : & £10 & : & £6 \end{array}$$

Find Two Equivalent Ratios

5:20

Multiply

$$\begin{array}{l} 5:20 \rightarrow \frac{5}{20} \\ \frac{5}{20} \cdot \frac{2}{2} = \frac{5 \cdot 2}{20 \cdot 2} = \frac{10}{40} \\ \frac{10}{40} \rightarrow 10:40 \end{array}$$

Divide

$$\begin{array}{l} 5:20 \rightarrow \frac{5}{20} \\ \frac{5}{20} \div \frac{5}{5} = \frac{5 \div 5}{20 \div 5} = \frac{1}{4} \\ \frac{1}{4} \rightarrow 1:4 \end{array}$$

Always remember

Ratios

A ratio is a way of comparing two or more quantities.

Purple paint is made by mixing **blue** and **red** paint in the ratio of **2 to 3**.



To make mortar, **sand** and **cement** are mixed together in the ratio of **5 to 2**.



Lilly, Jack and Jo have shared the money in the ratio of **2 to 6 to 3**.



A ratio must be written in the correct order, with the **quantity** mentioned first written first.

Ratios are easier to work out when they are in their simplest form. To simplify ratios, both numbers must be **divided by their highest common factor**.



The ratio of **blue** to **red** tiles is **6 to 3** but this can be simplified.

$$\begin{array}{c} 6:3 \\ \div 3 \quad \div 3 \\ \hline 2:1 \end{array}$$

3 is the highest common factor of 6 and 3, so divide both numbers by 3.

Dividing in a Ratio

Sometimes an amount needs to be divided according to a particular ratio. **Ava, Isla and Freya** made £315 selling balloons at a fayre. They agreed to split the money in the ratio of **3:2:4**. How much money does each person get?

1 Add the numbers in the ratio to calculate the total number of parts. $3 + 2 + 4 = 9$

2 Find the value of **1 part** by dividing the total amount by the total number of parts, 9. $315 \div 9 = 35$
1 part = 35

3 Multiply the value of **1 part**, 35, by the numbers in the ratio to calculate how much money each person gets. $3 \times 35 = 105$
 $2 \times 35 = 70$
 $4 \times 35 = 140$

4 315 divided in the ratio of **3:2:4** is **105:70:140**.
Check your answer by adding together the values. $105 + 70 + 140 = 315$

Assessment style question

Shannon is revising for her summer exams. The table below shows the number of minutes Shannon spends revising on each of 5 evenings. It also shows the number of minutes Shannon spends relaxing on the 5 evenings.

	Monday	Tuesday	Wednesday	Thursday	Friday
Number of minutes revising	88	198	150	133	160
Number of minutes relaxing	20	40	28	25	34

Sophie is making 400 scones. She uses butter, sugar and flour in the ratio 2:1:9. Here are the costs of those ingredients.

Butter	£2.20 per 500g
Sugar	£1.60 per kilogram
Flour	60p per 1.5kg

The total mass of the butter, sugar and flour in each scone is 30g

Work out the total cost of these ingredients for the 400 scones.

Mrs Chambers is organising a school trip to a museum for year 7 and year 8. She needs to work out the total cost of the museum tickets and bus hire. The table below shows the museum ticket prices.

Visitor Age	Price
0 - 3	free
4 - 12	£4.50
13 - 17	£6.50
18+	£11.50

Each bus has 51 seats and costs £125

Altogether 300 students want to go on the trip. The ratio of the number of students to the number of teachers is 25:1. The ratio of the number of students in year 7 to the number of students in year 8 is 8:7.

At the time of the trip, all of the students in year 7 are 11 or 12 years old. Of year 8 students, the ratio of number of 12 year olds to 13 year olds is 2:3. Work out the total price of the school trip.

Maths - Foundation

Real life Graphs

Key vocabulary

Coordinates - a set of value that show an exact position on a coordinate grid

Linear equation - an equation, when plotted, makes a straight line

Gradient - the steepness of the line of a linear equation

y-intercept - where the linear equation cuts the y-axis

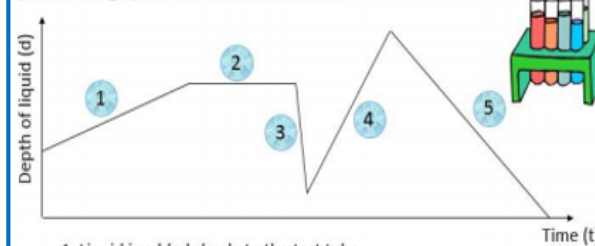
Substitution - when you replace an unknown for a given value

Picture perfect

Graphs can be used to represent a number of real life situations. It is important to read the labels on both axes to determine the meaning of the graph.

Example:

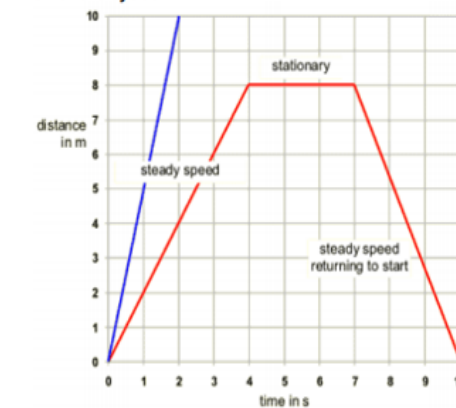
A test tube containing a chemical liquid is used in an experiment. During the experiment the **depth d** of the liquid changes with **time t**. Match the different parts of the graph to the statements below.



1. Liquid is added slowly to the test tube.
2. The level of the liquid remains constant.
3. Some liquid is poured out quickly.
4. Some liquid is poured in quite quickly.
5. The test tube is emptied.

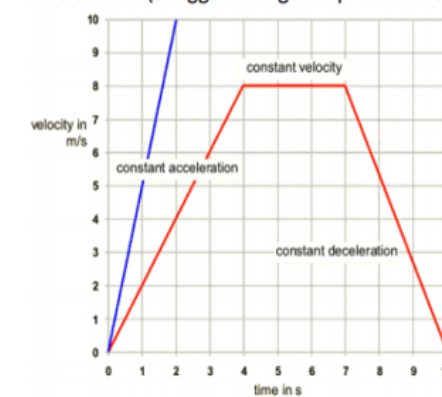
Always remember Distance-time graphs

Distance time graphs show distance away from a point. When an object is stationary, the line on the graph is horizontal. When an object is moving at a steady speed, the line on the graph is straight, but sloped. The **steeper** the line, the greater the **speed** of the object.



Speed-time graphs

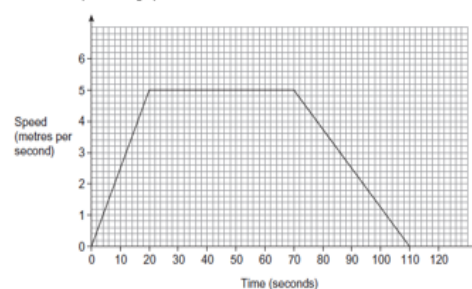
A speed-time graph tells us **changes** over **time**. When the object is travelling at a constant speed, the line on the graph is horizontal. When an object is accelerating or decelerating, the line on the graph is sloped. The **steeper** the gradient of the line, the greater the **acceleration** (a bigger change in speed in the same time).



Assessment style question

The distance around a cycle track is 400 metres.

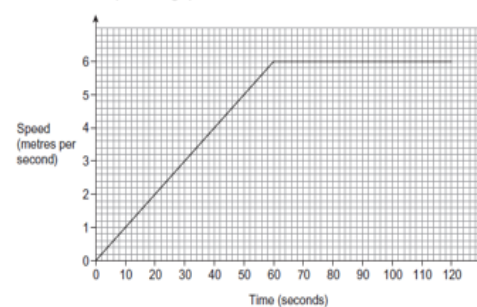
Robin cycles on the track. Here is his speed-time graph.



(a) Show that Robin cycles **exactly** once around the track in 110 seconds.

(b) Sanjay cycles on the same track.

Here is his speed-time graph.



Does Sanjay cycle the first 400 metres in a quicker time than Robin? You **must** show your working.

(3)

Maths - Foundation

Standard Form

Key vocabulary

- Standard form
- Ordinary number
- Power
- Index Laws
- Convert
- Ordinary number
- Adding, subtracting
- Multiplying, dividing

Picture perfect

Basic Structure

$$1 \leq a < 10 \leftarrow a \times 10^b \rightarrow \text{Whole number}$$

$$2.83 \times 10^6 = 2830000$$

Positive power of 10 = Large number

$$3.14 \times 10^{-4} = 0.000314$$

Negative power of 10 = Small decimal number

Standard Form

Positive Power = Large Number

$$4.3 \times 10^6 = 4\,300\,000$$

Negative Power = Small Number

$$2.1 \times 10^{-3} = 0.021$$

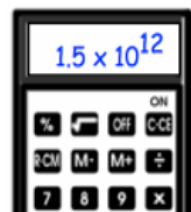
Ordinary Number	Standard Form
29	2.9×10^1
350	3.50×10^2
4716	4.716×10^3
600000000	6×10^8
0.3	3×10^{-1}
0.09	9×10^{-2}
0.0071	7.1×10^{-3}
0.000502	5.02×10^{-4}

Assessment style question Peter has multiplied two numbers using his calculator. The calculator shows the answer. He can remember that one number was 5000. What was the other number used in the multiplication?

The mass of Earth is 5.97×10^{24}

The mass of Jupiter is 1.898×10^{27}

Using a calculator, work out how many times heavier Jupiter is than Earth. Give your answer to one decimal place.



Always remember

A number is converted into **standard form** when the number is very large or very small, this mainly used in science and astronomy.

- The format of a number in standard form consists of a number between 1 and 10 **but cannot be 10**, multiplied by a power of 10.

$$(1 \leq x < 10) \times 10^n$$

- Converting a **very small number into standard form**: Size of a bacteria is 0.00000037 $0.00000037 = 3.7 \times 10^{-7}$
- Converting a **very large number into standard form**: Distance from Earth to the sun is 147100 million metres $147\,100\,000\,000 = 1.471 \times 10^{11}$
- Converting into a **small ordinary number** $2.4 \times 10^{-6} = 0.0000024$
- Converting into a **large ordinary number** $5.67 \times 10^9 = 5\,670\,000\,000$

Common mistakes:

- When not in standard form but in the same format as the number is not between $1 \leq x < 10$
(too big) $76.18 \times 10^6 = 7.618 \times 10^7$ and (too small) $0.12 \times 10^{-6} = 1.2 \times 10^{-7}$
When the number is getting smaller the power gets bigger, and when the number gets bigger the power gets smaller

Multiply/Divide Standard form

Separate the numbers and powers of 10.
Multiply/Divide numbers,
Apply laws of indices to power of 10s
Give answer in Standard form

$$(4.6 \times 10^4) \times (3 \times 10^3)$$

$$4.6 \times 3 \times 10^4 \times 10^3$$

$$13.8 \times 10^7 \quad \times$$

$$1.38 \times 10^8 \quad \checkmark$$

$$(1.56 \times 10^{-4}) \div (7.5 \times 10^{-7})$$

$$1.56 \div 7.5 \times 10^{-4} \div 10^{-7}$$

$$0.208 \times 10^3 \quad \times$$

$$2.08 \times 10^2 \quad \checkmark$$

Add/Subtract Standard form

Take numbers out of Standard form.

Add/Subtract values.

Convert answer back to Standard form.

$$(3.23 \times 10^4) + (8.2 \times 10^3)$$

$$= 32300 + 8200$$

$$= 40500$$

$$= 4.05 \times 10^4$$

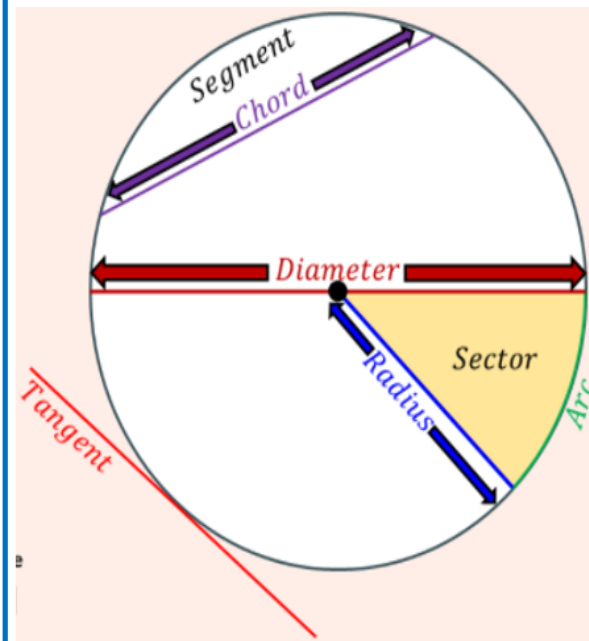
Maths - Higher

Circumference and area

Key vocabulary

Circumference
The perimeter <u>around</u> the circle
Diameter
The distance <u>across the centre</u> of the circle
Radius
The distance from the <u>centre to the edge</u> of the circle
Sector
<u>Part of the area</u> of a circle, enclosed by two radii
Arc
<u>Part of the circumference</u> of a circle
Tangent
A straight line that touches the curve of the circle at a point
Chord
A straight line segment between two points on the circle edge
Segment
The area created by the chord

Picture perfect

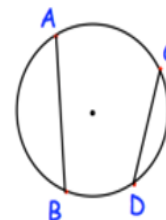


Assessment style question

Nicole is a wedding organiser.
The guests are to sit at circular tables with a diameter of 180cm.
Each guest needs 70cm around the circumference of the table.
There are 18 tables at the venue.
A total of 145 guests are attending the wedding
Are there enough tables?



- Draw a circle with two chords, AB and CD.
- Construct the perpendicular bisector of AB.
- Construct the perpendicular bisector of CD.
- What do you notice about where the two perpendicular bisectors meet?



Always remember

Area

$A = \pi r^2$ → Pi times the radius squared

Diameter is double the radius

$A = \pi \times 6.5^2$

$A = \pi \times 42.25$

$A = 132.73m^2$

Sector

Calculate the proportion of the circle required then use area formula

$A = \frac{n^\circ}{360} \pi r^2$

$\frac{85^\circ}{360^\circ} (\pi \times 6^2)$

$26.7cm^2$

Circumference

$C = \pi d$

$C = 2\pi r$

The circumference is always about three times the length of the diameter

$C = \pi \times 12cm$

$C = 37.7cm$

Arc length

Calculate the proportion of the circle required then use circumference formula

$L = \frac{n^\circ}{360} \pi d$

$\frac{85^\circ}{360^\circ} (\pi \times 12)$

$8.90cm$

Maths - Higher

Equations

Key vocabulary

Inverse: This is another word for opposite. We complete the opposite operation to the one shown in the question.

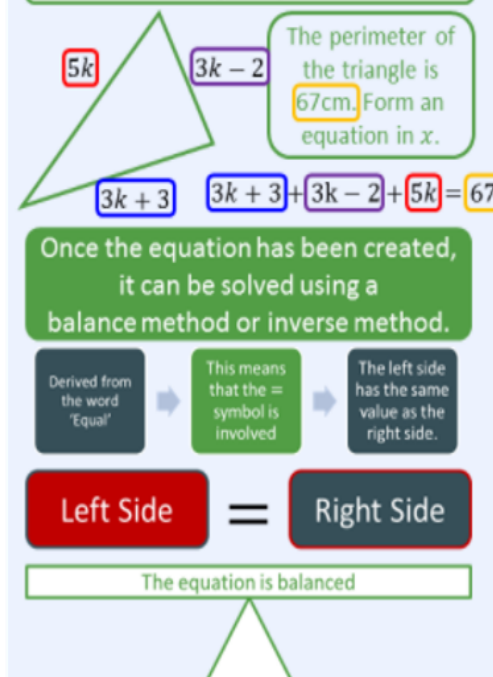
Integer: A whole number.

Equation: A mathematical statement that shows that two expressions are equal.

Solve: To get the solution or answer to a question.

Picture perfect

Creating equations



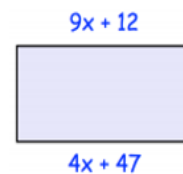
Assessment style question

Shown is a rectangle

(a) Explain why $9x + 12 = 4x + 47$

(b) Find x

Explain why $8x + 3 = 2(4x + 1)$ has no solution.



Spot the mistake:

Solve $7x - 5 = 5x + 23$

$$\begin{array}{r} -5x \quad -5x \\ 2x - 5 = 23 \\ -5 \quad -5 \\ 2x = 18 \\ \div 2 \quad \div 2 \\ x = 9 \end{array}$$

Always remember

As with all mathematical calculations, please remember to use

BIDMAS: Brackets then Indices then Division & Multiplication then Addition & Subtraction

Solving linear equations

General 4 step process

- Expand brackets and simplify (collect like terms)
- If x is on both sides, eliminate smallest value
- Eliminate excess number
- Divide and solve for x



$$3(x + 1) = 2(x + 2)$$

$$\begin{array}{r} 3x + 3 = 2x + 4 \\ -2x \quad -2x \\ x + 3 = 4 \\ -3 \quad -3 \\ x = 1 \end{array}$$

Advanced equations

Equations where fractions are involved

Fractions are divisions and can be eliminated by multiplying

$$\frac{x}{2} = 5 \quad x = 10$$

$$\times 2 \quad \times 2$$

Remove variable from denominator

$$\frac{2y}{(3-y)} = 4 \rightarrow 2y = 4(3-y)$$

$$\times (3-y) \quad \times (3-y)$$

Cross-multiplying allows us to move terms in a fraction from one side of an equation to the other

$$\frac{x+1}{3} = \frac{x}{2} \rightarrow 2(x+1) = 3x$$

An equation with TWO UNKNOWNNS

$$\begin{array}{r} -3x \quad -1 \quad -3x \quad -1 \\ 5x + 1 = 3x + 17 \\ +2 \quad 2x = 16 \div 2 \\ x = 8 \end{array}$$

Maths - Higher

Indices

Key vocabulary

Square: A square number is the result of multiplying a number by itself.

Cube: A cube number is the result of multiplying a number by itself twice.

Root: A root is the reverse of a power.

Indices: These are the squares, cubes and powers.

Operation: In maths these are the functions $\square \square + -$.

Picture perfect

indices

$$a^0 = 1 \quad a^{m/n} = (\sqrt[n]{a})^m$$

$$a^{-n} = \frac{1}{a^n} \quad a^{-1} = \frac{1}{a}$$

$$a^{1/n} = \sqrt[n]{a} \quad a^{1/2} = \sqrt{a}$$

Assessment style question

Question 1: Can you spot any mistakes?

$$6^2 = 12$$

$$1^7 = 7$$

$$2^6 \times 2^3 = 4^9$$

$$7^{15} \div 7^5 = 7^3$$

$$10^4 = 40$$

$$2^6 = 32$$

$$6^3 \times 6^4 = 6^{12}$$

Question 1: Can you spot any mistakes?

Always remember

Basic Laws of Indices

Special indices to consider

$x^1 = x$	Anything to the power 1 = itself
$x^0 = 1$	Anything to the power 0 = 1
$1^x = 1$	1 to the power of anything = 1

These laws can be applied if the bases are the same

$x^a \times x^b = x^{a+b}$ $z^3 \times z^7 = z^{10}$	When multiplying powers with the same base – Add the powers
$x^a \div x^b = x^{a-b}$ $s^2 \div s^5 = s^{-3}$	When dividing powers with the same base – Subtract the powers
$(x^a)^b = x^{a \times b}$ $(e^4)^3 = e^{12}$	When raising the power (brackets) – Multiply the powers

Advanced Laws of Indices

Negative Indices

$$x^{-n} \rightarrow \frac{1}{x^n}$$

Find Reciprocal
Apply Positive Power
Apply top and bottom

$$z^{-3} = \left(\frac{1}{z}\right)^3 = \frac{1}{z^3} \quad 6^{-2} = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

Fractional Indices

$$x^{\frac{m}{n}} = (\sqrt[n]{x})^m$$

Root by denominator first
Then power of numerator

$$\frac{1}{x^2} = \sqrt{x} \quad \frac{1}{x^3} = \sqrt[3]{x} \quad \frac{1}{x^4} = \sqrt[4]{x} \quad x^{\frac{2}{3}} = (\sqrt[3]{x})^2$$

$$64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = (4)^2 = 16$$

Negative Fractional Indices

$$x^{-\frac{a}{b}} = \frac{1}{(\sqrt[b]{x})^a}$$

Negative Fractional Powers:
Apply reciprocal first!

$$9^{-\frac{3}{2}} = \frac{1}{9^{\frac{3}{2}}} = \frac{1}{(\sqrt{9})^3} = \frac{1}{(3)^3}$$

Maths - Higher

Measures

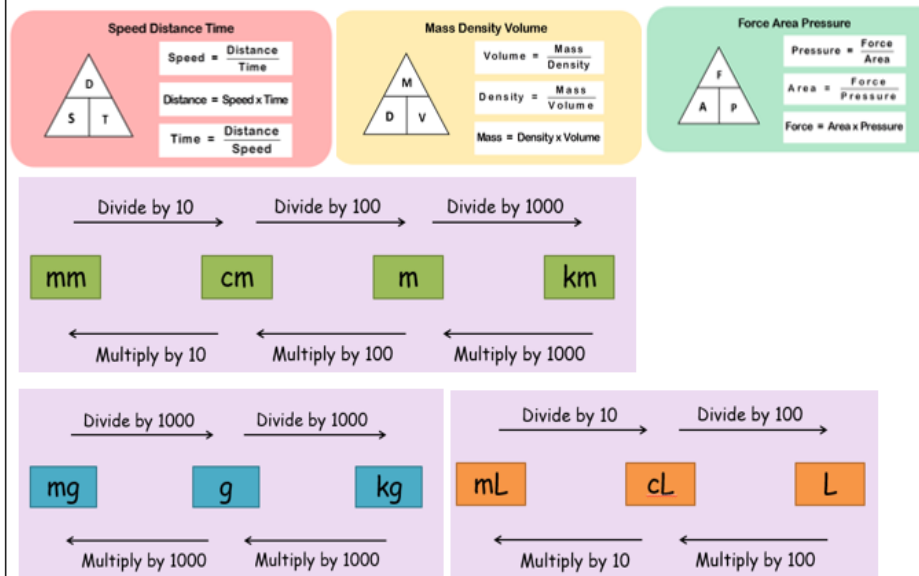
Key vocabulary

Metric, imperial, speed, density, conversion, length, capacity, mass, upper and lower bounds, limits of accuracy, error interval

Next Steps

- Area and volume conversions

Picture perfect



Always remember

Length	Mass	Capacity
1 cm = 10 mm	1 g = 1000 mg	1 cl = 10 ml
1 m = 100 cm	1 kg = 1000 g	1 cm ³ = 1 ml
1 km = 1000 m	1 tonne = 1000 kg	1 litre = 1000 ml
		1 litre = 1000 cm ³

Upper and Lower Bounds

Any recorded measurement has almost certainly been rounded. The true value will be somewhere between the lower and upper bound.

Lower bound = smallest possible number that rounds up to the given number.

Upper bound = largest possible number that rounds down to the given number.

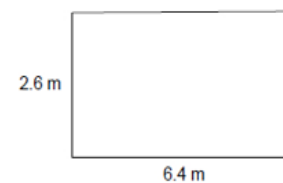
The lower and upper bounds are sometimes known as limits of accuracy and the range between them is the error interval.

Assessment style question:

Tom's car travels 40 miles per gallon.
One litre of petrol costs £1.19
1 gallon = 4.5 litres

Work out the cost of petrol when Tom drives 200 miles.

The dimensions of a rectangular floor are to the nearest 0.1 metres.



A force of 345 Newtons is applied to the floor.
The force is to the nearest 5 Newtons.

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the upper bound of the pressure.
Give your answer to 4 significant figures.
You **must** show your working.

Maths - Higher

Perimeter and Area

Key vocabulary

Perimeter- The length around a shape

Area- The size within a shape

Surface Area- The total areas of each face of a 3D shape

Regular- All the sides and angles of a shape are equal

Perpendicular height- The height that forms a right angle with the base length.

Face- The flat surface of a 3D shape

Edge- The line where two faces meet

Vertex- Corner of a shape

Prism- A 3D shape that has the same face when you cut it along its length. Eg: a cuboid, a loaf of bread.

Cross section- The constant face of a prism. Eg: for a cylinder its cross section is a circle.

Picture perfect

Perimeter

The total distance **AROUND** a 2D shape

Adding all the side lengths together

$$\begin{array}{|c|c|} \hline 100m & 100 + 100 + 35 + 35 \\ \hline 35m & 270m \\ \hline 100m & \end{array}$$

The process does not change if we have algebraic terms

$$\begin{array}{c} x \quad y \quad x \quad x \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ 2x \quad 3x \quad 2x-y \end{array} = 9x + 2y$$

Rectangular areas

The total **space** taken up by a 2D shape

Multiplying two side lengths together

Area of rectangle $\square = l \times w$

$$\begin{array}{|c|c|} \hline 12cm & \text{Area} = 6cm \times 12cm \\ \hline 6cm & 72cm^2 \\ \hline \end{array}$$

Always remember

Triangular areas

The area of a triangle takes up **half** the space of the rectangle that is formed around it

$$\text{Area of triangle} \triangle = \frac{1}{2}(b \times h)$$

$$\begin{array}{c} 4m \\ \diagup \quad \diagdown \\ 7m \end{array} \quad A = \frac{1}{2}(7m \times 4m) = \frac{1}{2}(28m^2) = 14m^2$$

Be sure to use perpendicular heights

Calculate base \times height first

Remember to **halve** your answer!

Rectangular areas

With compound shapes, break it down.

$$\begin{array}{|c|c|c|} \hline 5cm & 5cm & 5cm \\ \hline A & B & C \\ \hline 5cm & 5cm & 12cm \\ \hline 3cm & & \end{array} \quad \begin{array}{l} A = 60cm^2 \\ B = 20cm^2 \\ C = 60cm^2 \\ \hline 140cm^2 \end{array}$$

Parallelogram

Imagine a tilted rectangle

$$\begin{array}{c} \diagup \quad \diagdown \\ b \quad h \end{array} \quad \square = b \times h$$

Be sure to use **perpendicular heights**

Trapezium

A more complex formula to know

$$\text{Trapezium} = \frac{1}{2}(a + b) \times h$$

$$\begin{array}{c} a \\ \diagdown \quad \diagup \\ b \quad h \end{array} \quad \begin{array}{l} \text{Add the parallel sides} \\ \text{Halve it} \\ \text{Multiply by height} \end{array}$$

Assessment style question

A cube has a volume of $27cm^3$ and a surface area $36cm^2$.

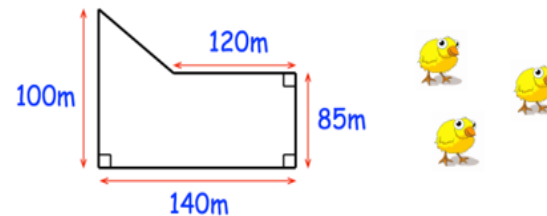
How long is each side?

A cube has a volume of $8cm^3$

What is the surface area?

A cube has a surface area of $6cm^2$. What is its area?

Question 2: Farmer Martin keeps chickens in the field below. Each chicken needs $3m^2$. What is the maximum number of chickens that he can keep?



Maths - Higher

Properties of polygons

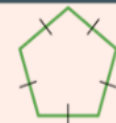
Key vocabulary

Picture perfect

Regular

Side lengths are the same

Interior and exterior angles the same



Irregular

Side lengths are not ALL the same

Interior and exterior angles the not ALL the same



Equal sides are marked with a dash through the line

Convex

The shape is 'bulging' outwards

All angles less than 180°



Concave

The shape has 'caved' inwards

One or more angles is greater than 180°

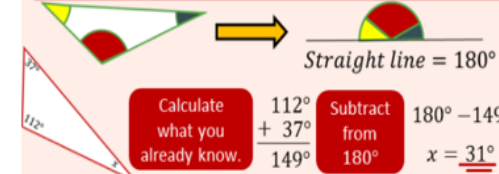


Assessment style question

Always remember

Triangles

All three angles can be orientated to fit on a straight line \rightarrow All angles in a triangle make 180°



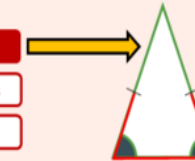
Exterior angle = Sum of two angles on opposite side.



Isosceles triangle

It has two equal lengths

It has two equal angles



They are classified by the number of sides they have

Number of sides	Name of shape
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Polygons

Knowledge of triangles is important

Number of sides	Number of \triangle	Sum of interior angles	Regular interior angle	Regular exterior angle
3	1	180°	60°	120°
4	2	360°	90°	90°
5	3	540°	108°	72°
6	4	720°	120°	60°
7	5	900°	129°	51°
8	6	1080°	135°	45°
n	$(n - 2)$	$(n - 2) \times 180^\circ$	$\frac{(n - 2) \times 180^\circ}{n}$	$360^\circ \div n$

The number of triangles in a shape will always be **TWO** less than the number of sides



Maths - Higher

Ratio and Proportion

Key vocabulary

Ratio: Relationship between two or more numbers.
Part: This is the numeric value '1' of, would be equivalent to.
Simplify: Divide all parts of a ratio by the same number.
Equivalent: Equal in value.
Convert: Change from one form to another.
Scale: The ratio of the length in a drawing to the length of the real thing.
Proportion: A name we give to a statement that two ratios are equal.
Exchange rate: The value of one currency for the purpose of conversion to another.

Assessment style question

Sophie is making 400 scones. She uses butter, sugar and flour in the ratio 2:1:9. Here are the costs of those ingredients.

Butter	£2.20 per 500g
Sugar	£1.60 per kilogram
Flour	60p per 1.5kg

The total mass of the butter, sugar and flour in each scone is 30g

Work out the total cost of these ingredients for the 400 scones.

James is making concrete using cement, sand and gravel in the ratio 1 : 2 : 3
 James has:
 63kg cement
 112kg sand
 210kg gravel

What is the maximum amount of concrete that James can make?

Picture perfect

Ratio: The is the relationship between two or more numbers and each number is separate by a colon.



The ratio of footballs to rugby balls: 1:4

The ratio of rugby balls to footballs: 4:1

Football is mentioned first so that is why the 1 comes before 4.

Rugby is mentioned first so that is why the 4 comes before 1.

Always remember

Is the relationship between two or more quantities

It is written in the form $a : b$

Compares one part to another part



The ratio of red to blue is 4 : 5

The ratio of blue to red is 5 : 4

Sentence structure is important!

Simplifying Ratios

$12 : 8 \xrightarrow{\div 4} 6 : 4 \xrightarrow{\div 2} 3 : 2$

Divide all numbers by the same value

The ratio of boys to girls in a Geography class is 15 : 5

What fraction of the class is girls?

$\frac{5 \text{ girls}}{20 \text{ total parts}} \Rightarrow \frac{1}{4}$

Direct Proportion

As one value increases, the other increases at the same rate

Three Coffees cost £7.50,

How much would five Coffees cost?

Find the value of one coffee then multiply by quantity needed

$£7.50 \div 3 = £2.50 \text{ per coffee}$

$£2.50 \times 5 = £12.50$

Inverse Proportion

As one value increases, the other decreases at the same rate

It takes 3 men 4 days to build a wall.

How long would it take 2 men?

Find the time taken by one man then divide by quantity stated

$3 \text{ men} \times 4 \text{ days} = 12 \text{ days}$

$12 \text{ days} \div 2 \text{ men} = 6 \text{ days}$

Sharing in a given ratio

Find total number of parts → Find value of one part → Multiply by original ratio

Share \$40 in the ratio 3 : 5

Find total number of parts → Add the ratio parts together

$3 + 5 = 8$

Find value of one part → Divide amount by number of parts

$\$40 \div 8 = \5

Each part of the ratio is worth \$5

Multiply by original ratio

$3 : 5 \xrightarrow{\times \$5} \$15 : \25

Mark and John have sweets in the ratio 3 : 4. If Mark has 27 sweets. How many does John have?

$27 \div 3 = 9 \text{ sweets per part}$

$4 \times 9 = 36 \text{ (John's sweets)}$

Map scale factors

It is the ratio of a distance on the map/model to the corresponding size in real life.

Written in the form $1 : n$

Map or Model $\xrightarrow{\times \text{scale factor}}$ Real life
 Real life $\xrightarrow{\div \text{scale factor}}$ Map or Model

Know your conversions

$10\text{mm} = 1\text{cm}$
 $100\text{cm} = 1\text{m}$
 $1000\text{m} = 1\text{km}$

A map has a scale of 1:25000.

Michael is 6cm from his home.

How far from home is he?

Give your answer in km

$6\text{cm} \times 25000 = 150000\text{cm}$
 $150000\text{cm} \xrightarrow{\div 100} 1500\text{m} \xrightarrow{\div 1000} 1.5\text{km}$

Maths - Higher

Real Life Graphs

Key vocabulary

Graph
Real life
Distance
Time
Depth/water level
Money
Interpret
Draw
Describe

Picture perfect

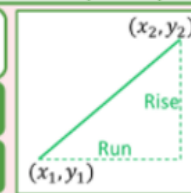
Rate of Change

A rate that describes how one quantity changes in relation to another quantity

It is represented by the Gradient of a line

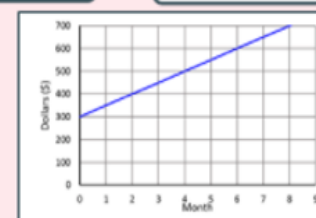
$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}}$$



Interpreting Rates of Change

Gradient → Amount of (y) per Amount of (x)



Rate of change = \$50 per month

Assessment style question

A conversion graph to convert between Euros and US Dollars.

Horizontal axis: Euros from 0 to €100

Vertical axis: US Dollar (decide scale yourself)

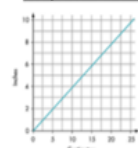
US Dollars	\$ 77
Euros	€ 70

Example 2: Using the graph below, identify what A, B and C mean in terms of travel.



A = steady speed,
B = no movement,
C = steady speed back to start

Using a conversion graph

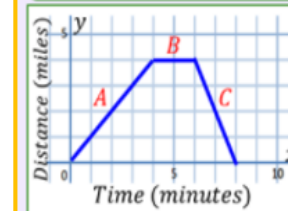


Conversion graphs can be used to convert between any 2 units which have a linear relationship. Here, you can use the graph to convert between inches and centimetres

Always remember

Distance – Time graphs

Distance - Time graphs record the journey of an object as it begins to move away from and return to a point.



A Moving away
B Stationary
C Returning

Gradient = Speed

Not all objects travel at a constant speed



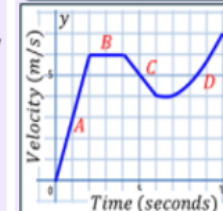
$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Calculate speed at a specific point by creating a tangent.

Velocity – Time graphs

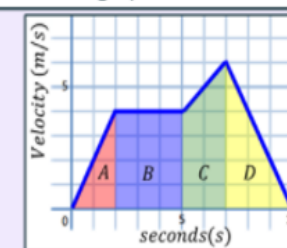
Velocity - Time graphs record the velocity of a particle moving along a straight line



Stage A
Constant rate of acceleration
Stage B
Steady 'constant' speed
Stage C
Constant rate of deceleration
Stage D
Increasing rate of acceleration

The gradient of the line is the acceleration.

Area under graph = Distance travelled

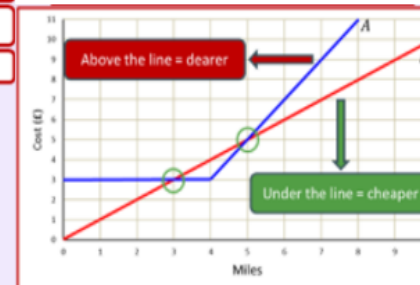
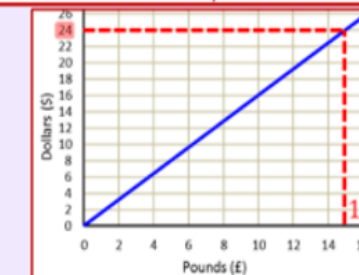


Financial graphs

Currency Conversions

Predict future costs

Cost Comparisons



Maths - Higher

Standard Form

Key vocabulary

Indices
Power
Power of ten
Standard form
Ordinary number
Convert

Picture perfect

Basic Structure

$$1 \leq a < 10 \leftarrow a \times 10^b \rightarrow \text{Whole number}$$

$$2.83 \times 10^6 = 2830000$$

Positive power of 10 = Large number

$$3.14 \times 10^{-4} = 0.000314$$

Negative power of 10 = Small decimal number

Powers of 10:

$$10^6 = 1000000$$

$$10^3 = 1000$$

$$10^2 = 100$$

$$10^1 = 10$$

$$10^0 = 1$$

$$10^{-1} = 0.1$$

$$10^{-2} = 0.01$$

$$10^{-3} = 0.001$$

$$10^{-6} = 0.000001$$

Standard Form

Positive Power = Large Number

$$4.3 \times 10^6 = 4\,300\,000$$

Negative Power = Small Number

$$2.1 \times 10^{-3} = 0.021$$

Always remember

Add/Subtract Standard form

Take numbers out of Standard form.
Add/Subtract values.
Convert answer back to Standard form.

$$\begin{aligned} & (3.23 \times 10^4) + (8.2 \times 10^3) \\ &= 32300 + 8200 \\ &= 40500 \\ &= \underline{\underline{4.05 \times 10^4}} \end{aligned}$$

Multiply/Divide Standard form

Separate the numbers and powers of 10.
Multiply/Divide numbers,
Apply laws of indices to power of 10s
Give answer in Standard form

$$\begin{aligned} & (4.6 \times 10^4) \times (3 \times 10^3) \\ & \quad \boxed{4.6 \times 3} \times \boxed{10^4 \times 10^3} \\ & \quad 13.8 \times 10^7 \quad \times \\ & \quad \underline{\underline{1.38 \times 10^8}} \quad \checkmark \end{aligned}$$

$$\begin{aligned} & (1.56 \times 10^{-4}) \div (7.5 \times 10^{-7}) \\ & \quad \boxed{1.56 \div 7.5} \times \boxed{10^{-4} \div 10^{-7}} \\ & \quad 0.208 \times 10^3 \quad \times \\ & \quad \underline{\underline{2.08 \times 10^2}} \quad \checkmark \end{aligned}$$

Assessment style question

Here are five numbers.

47 000 4.5×10^4 5×10^3 2.8×10^5 125 000

Work out the difference between the largest and smallest numbers.

Give your answer in standard form.

Solve $\frac{x}{0.02} = 3.1 \times 10^{-4}$

Give your answer in standard form.

Maths - Higher

Surds

Key vocabulary

Indices: The number of times a number is multiplied by itself.

Roots: - Square Root - Cube Root

Surds: Surds are numbers left in 'square root form' (or 'cube root form' etc). They are therefore irrational numbers. The reason we leave them as surds is because in decimal form they would go on forever.

Rationalise: The process by which a fraction is rewritten so that the denominator contains only rational numbers, i.e. no roots.

Picture perfect

Law of Surds

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$
- $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
- $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$
- $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

Always remember

Surds are expressions which contain an irrational square root

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b} \quad \sqrt{3} \times \sqrt{7} = \sqrt{3 \times 7} = \sqrt{21}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \quad \frac{\sqrt{6}}{\sqrt{10}} = \sqrt{\frac{6^3}{10^5}} = \sqrt{\frac{3}{5}}$$

$$\sqrt{a} + \sqrt{b} \neq \sqrt{a+b} \quad \sqrt{5} + \sqrt{20} = \sqrt{25} \times$$

Writing in the form $a\sqrt{b}$

Think square numbers $\sqrt{200}$ Square Factors = 4, 25, 100
Choose the largest square factor
 $\sqrt{100} \times \sqrt{2} = 10\sqrt{2}$

Rationalising the denominator

Rationalising the denominator involves removing all of the roots from the bottom of a fraction.

$$\frac{6}{\sqrt{3}} \Rightarrow \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \quad \text{Multiply top and bottom by irrational root} \Rightarrow \frac{6\sqrt{3}}{\sqrt{9}} \Rightarrow \frac{6\sqrt{3}}{3}$$

A more complex denominator

$$\frac{5}{3 + \sqrt{2}} \Rightarrow \frac{5}{3 + \sqrt{2}} \times \frac{3 - \sqrt{2}}{3 - \sqrt{2}} \quad \text{Multiply top and bottom by Conjugate (opposite root)}$$

$$= \frac{5(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} \quad \text{Expand and simplify}$$

$$= \frac{15 - 5\sqrt{2}}{9 - 3\sqrt{2} + 3\sqrt{2} - 2} = \frac{15 - 5\sqrt{2}}{7}$$

Assessment style question

Question 1: Find the area of each of these rectangles

- (a) $\sqrt{10}$ cm $\sqrt{5}$ cm (b) $\sqrt{12}$ cm $\sqrt{3}$ cm (c) $\frac{9}{2}$ cm $2\sqrt{2}$ cm

Question 2: Find the perimeter of each of these rectangles

- (a) $\frac{10}{3}$ cm $\frac{5}{3}$ cm (b) $\sqrt{10} + \sqrt{2}$ cm $\sqrt{10} - \sqrt{2}$ cm (c) $\sqrt{72}$ cm $\sqrt{18}$ cm

Mrs Jenkins is making decorations for a wedding. She needs $18\sqrt{5}$ metres of ribbon in total. Mrs Jenkins has 40 metres of ribbon. Does she have enough ribbon?



Maths - Higher

Standard Form

Key vocabulary

Indices
Power
Power of ten
Standard form
Ordinary number
Convert

Assessment style question

Here are five numbers.

47 000 4.5×10^4 5×10^3 2.8×10^5 125 000

Work out the difference between the largest and smallest numbers.

Give your answer in standard form.

Picture perfect

Basic Structure

$$1 \leq a < 10 \quad \leftarrow a \times 10^b \quad \rightarrow \text{Whole number}$$

$$2.83 \times 10^6 = 2830000$$

Positive power of 10 = Large number

$$3.14 \times 10^{-4} = 0.000314$$

Negative power of 10 = Small decimal number

Powers of 10:

$$10^6 = 1000000$$

$$10^3 = 1000$$

$$10^2 = 100$$

$$10^1 = 10$$

$$10^0 = 1$$

$$10^{-1} = 0.1$$

$$10^{-2} = 0.01$$

$$10^{-3} = 0.001$$

$$10^{-6} = 0.000001$$

Standard Form

Positive Power = Large Number

$$4.3 \times 10^6 = 4\,300\,000$$

Negative Power = Small Number

$$2.1 \times 10^{-3} = 0.021$$

Always remember

Add/Subtract Standard form

Take numbers out of Standard form.

Add/Subtract values.

Convert answer back to Standard form.

$$(3.23 \times 10^4) + (8.2 \times 10^3)$$

$$= 32300 + 8200$$

$$= 40500$$

$$= 4.05 \times 10^4$$

Multiply/Divide Standard form

Separate the numbers and powers of 10.

Multiply/Divide numbers,

Apply laws of indices to power of 10s

Give answer in Standard form

$$(4.6 \times 10^4) \times (3 \times 10^3)$$

$$4.6 \times 3 \times 10^4 \times 10^3$$

$$13.8 \times 10^7 \quad \times$$

$$1.38 \times 10^8 \quad \checkmark$$

$$(1.56 \times 10^{-4}) \div (7.5 \times 10^{-7})$$

$$1.56 \div 7.5 \times 10^{-4} \div 10^{-7}$$

$$0.208 \times 10^3 \quad \times$$

$$2.08 \times 10^2 \quad \checkmark$$

Maths - Higher

Surds

Key vocabulary

Indices: The number of times a number is multiplied by itself.

Roots: - Square Root - Cube Root

Surds: Surds are numbers left in 'square root form' (or 'cube root form' etc). They are therefore irrational numbers. The reason we leave them as surds is because in decimal form they would go on forever.

Rationalise: The process by which a fraction is rewritten so that the denominator contains only rational numbers, i.e. no roots.

Picture perfect

Law of Surds

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$
- $a\sqrt{b} \times c\sqrt{d} = ac\sqrt{bd}$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
- $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$
- $\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$

Always remember

Surds are expressions which contain an irrational square root

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